Modul #11

TE3113
SISTEM KOMUNIKASI 1

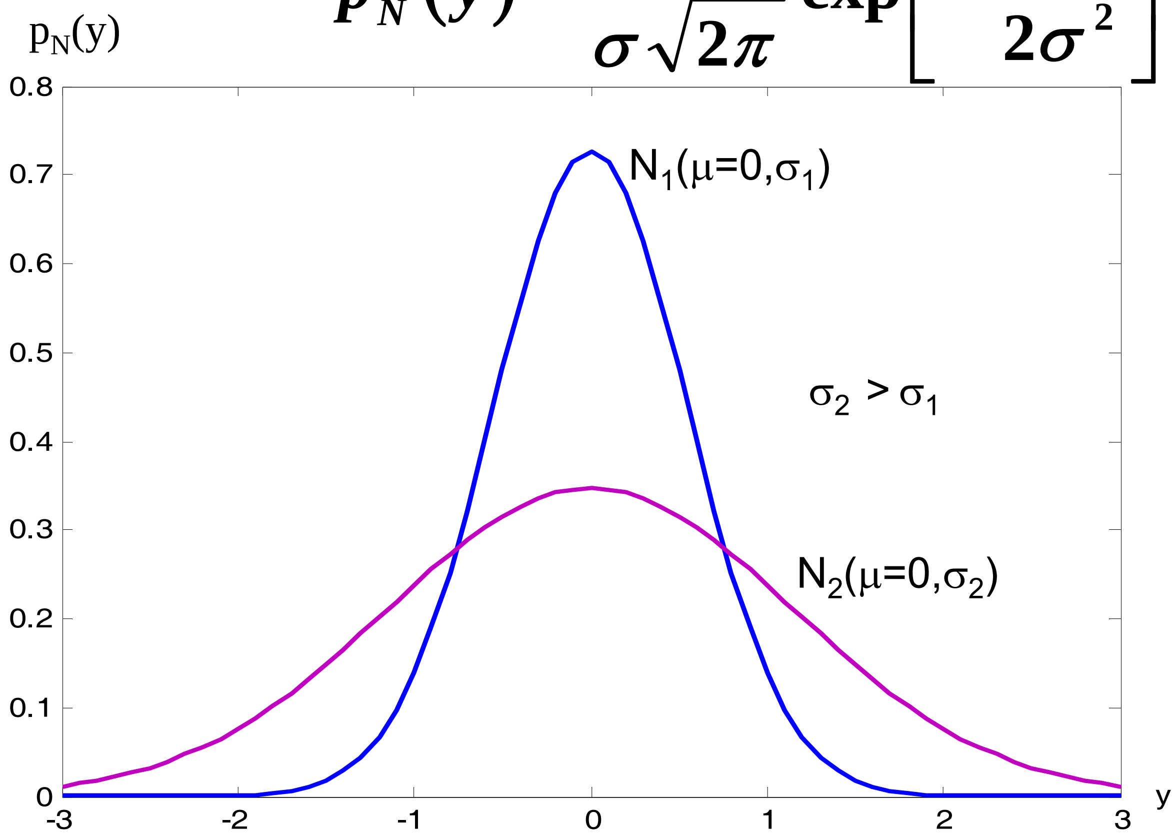
TRANSMISI BASE-BAND

Program Studi S1 Teknik Telekomunikasi
Departemen Teknik Elektro - Sekolah Tinggi Teknologi Telkom
Bandung – 2007

Gaussian/Normal

- **Normal Distribution:** Completely characterized by mean (μ) and variance (σ^2)

$$p_N(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{y^2}{2\sigma^2}\right]$$



Gaussian: Rapidly Dropping Tail Probability!

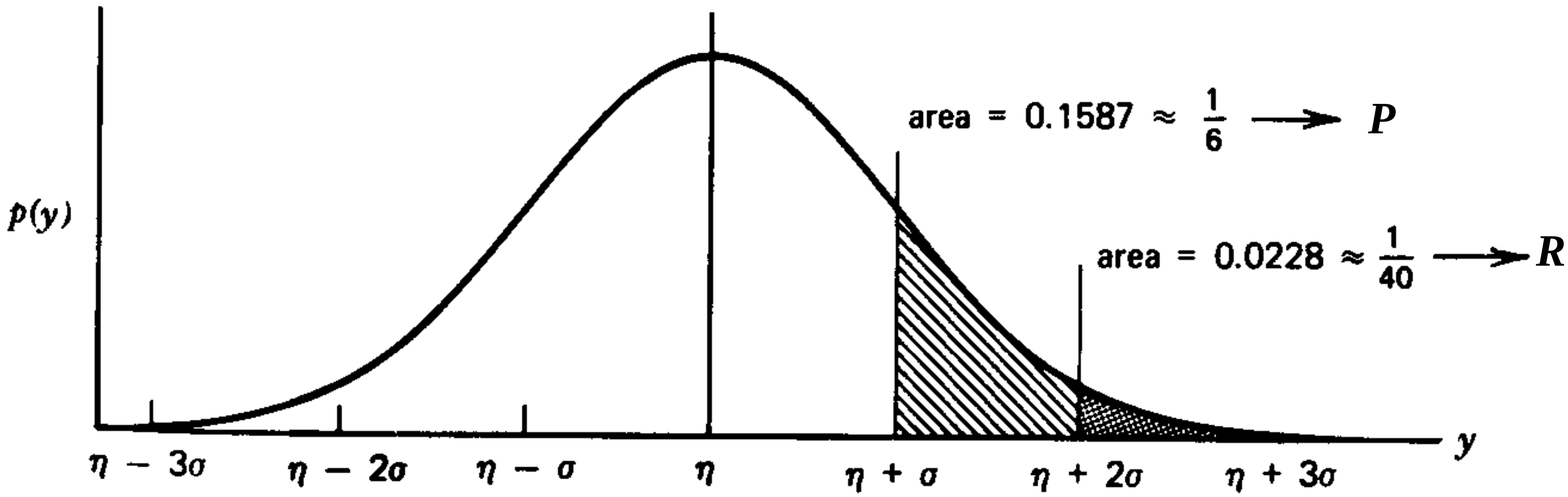


FIGURE 2.12. Tail areas of the normal distribution.

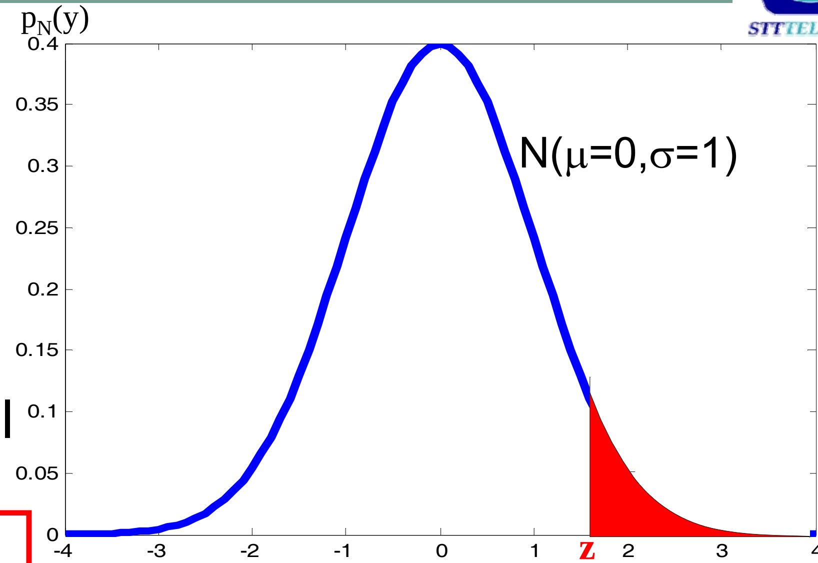
$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{\eta+\sigma}^{\infty} \exp\left[-\frac{(y-\eta)^2}{2\sigma^2}\right] dy$$

$$R = \frac{1}{\sigma\sqrt{2\pi}} \int_{\eta+2\sigma}^{\infty} \exp\left[-\frac{(y-\eta)^2}{2\sigma^2}\right] dy$$

Review Probabilitas dan Statistik

Gaussian/Normal

- **Normal Distribution:** Completely characterized by mean (μ=0) and variance (σ^2=1)

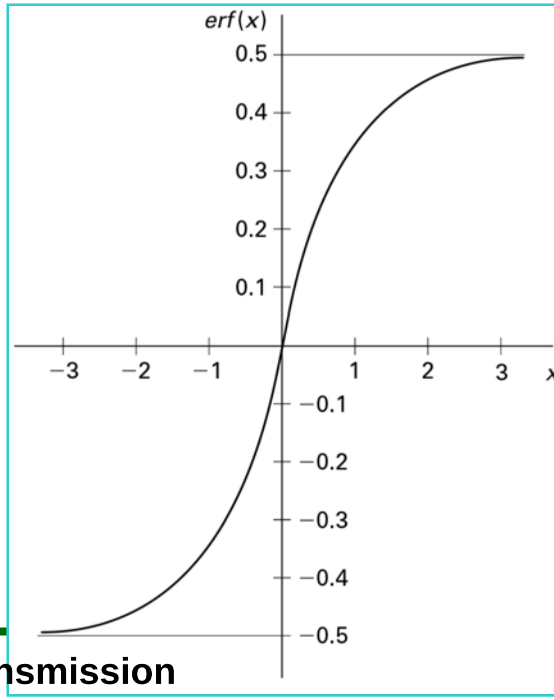


- **Q-function:** one-sided tail of normal pdf

$$Q(z) \triangleq p(y > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$

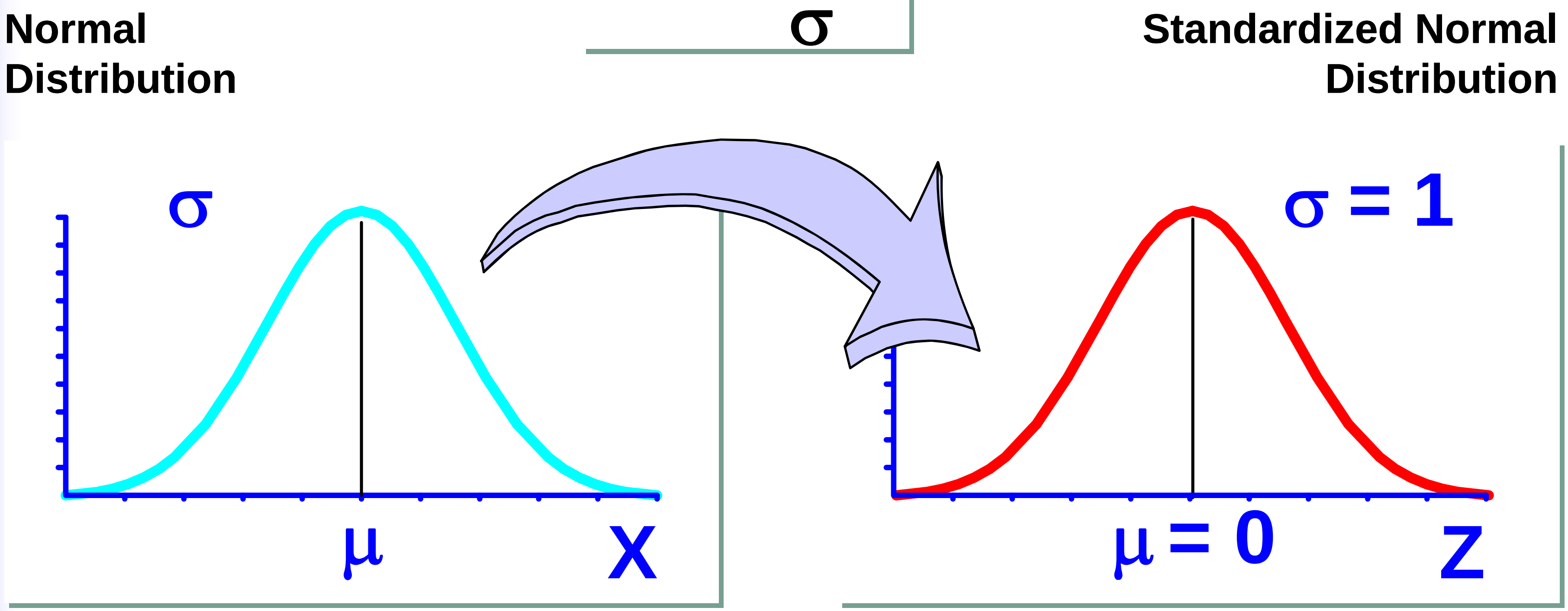
- **erfc():** two-sided tail.
- So:

$$Q(z) = \frac{1}{2} \text{erfc}\left(\frac{z}{\sqrt{2}}\right).$$



Standardize the Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$



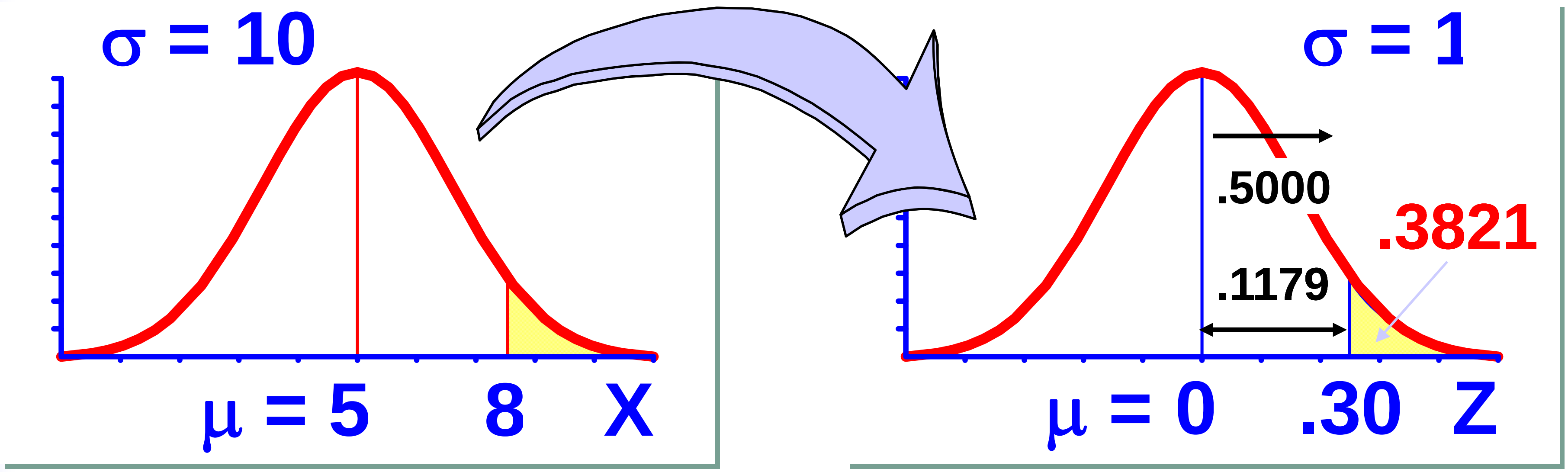
One table!

Example $P(X \geq 8)$

Normal Distribution

$$Z = \frac{X - \mu}{\sigma} = \frac{8 - 5}{10} = .30$$

Standardized Normal Distribution



Shaded area exaggerated

PERTANYAAN ! Luas daerah yang diarsir = 0.3821 = Q(??); ?? = **0.3**

Q-function: Tail of Normal Distribution

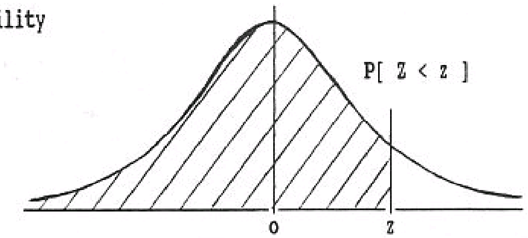
$Q(z) = P(Z > z) = 1 – P[Z < z]$

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

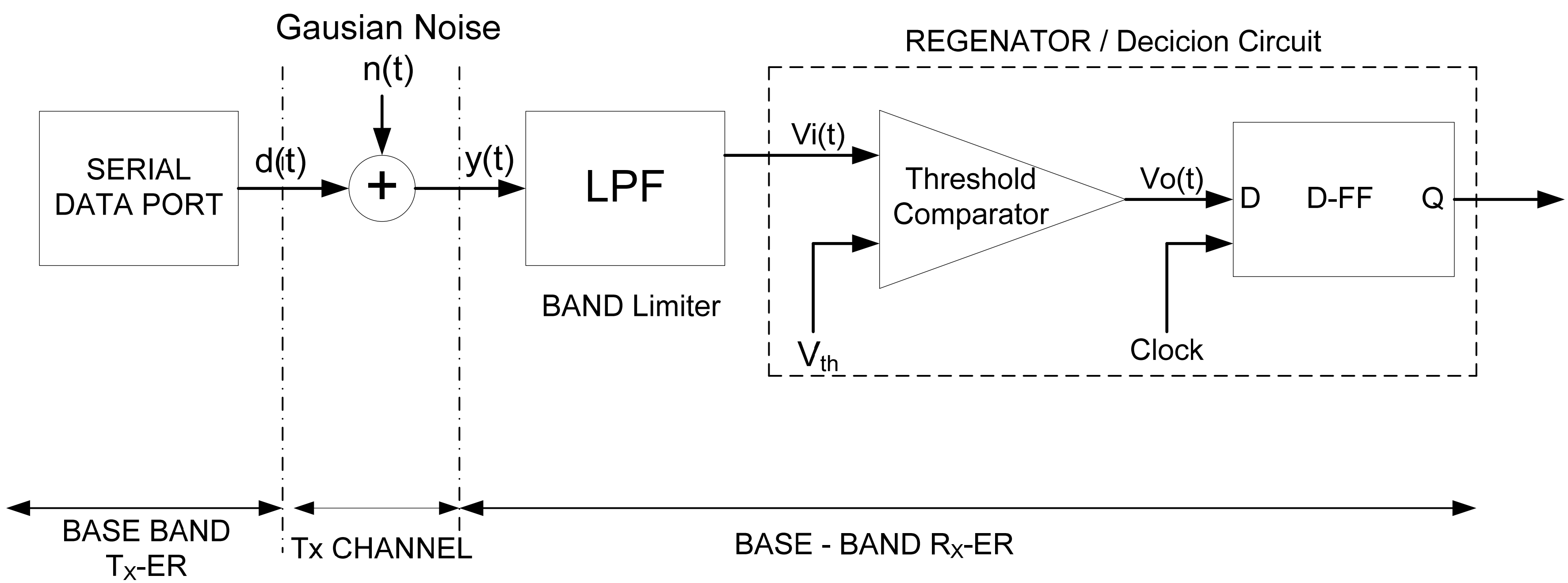
$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Baseband Digital Transmission Link



Sinyal Terima + AWGN

original message
d(t)

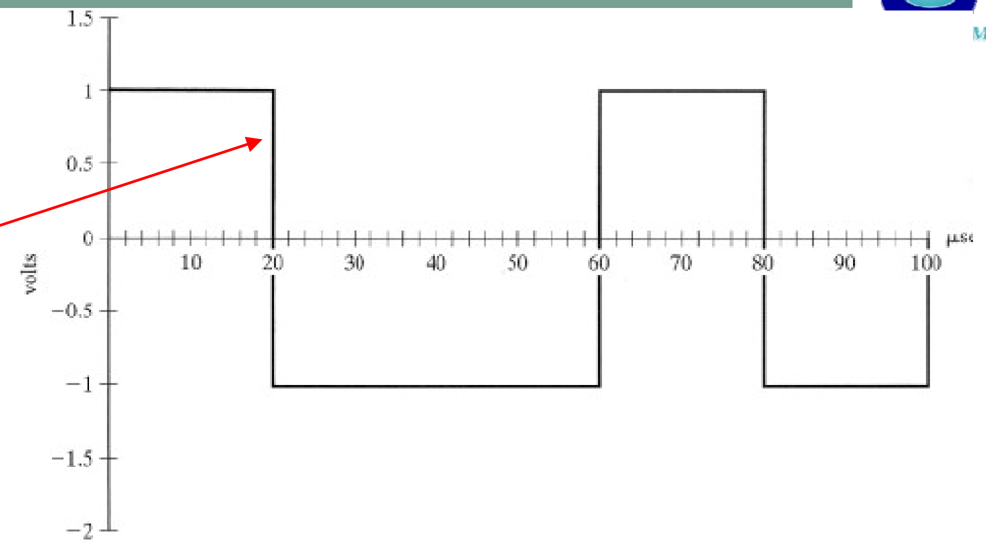
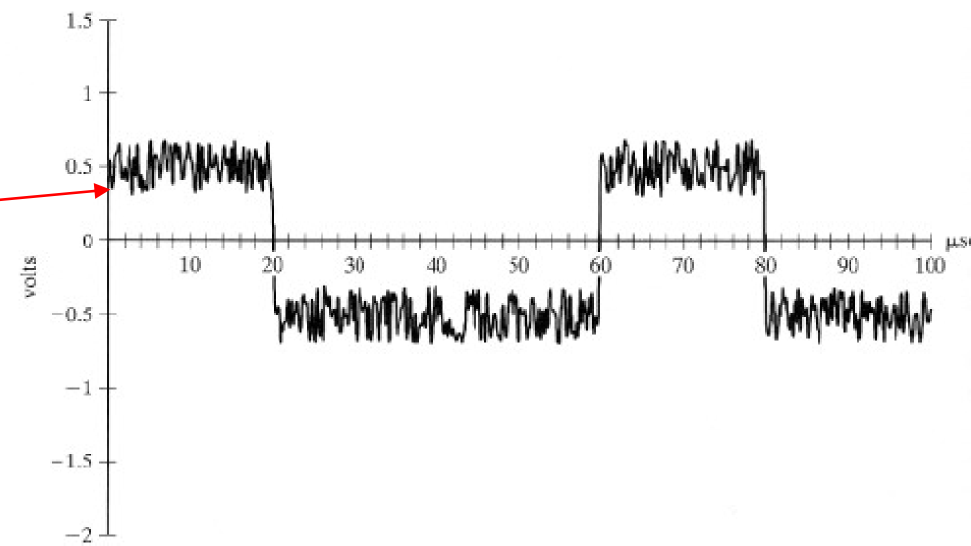


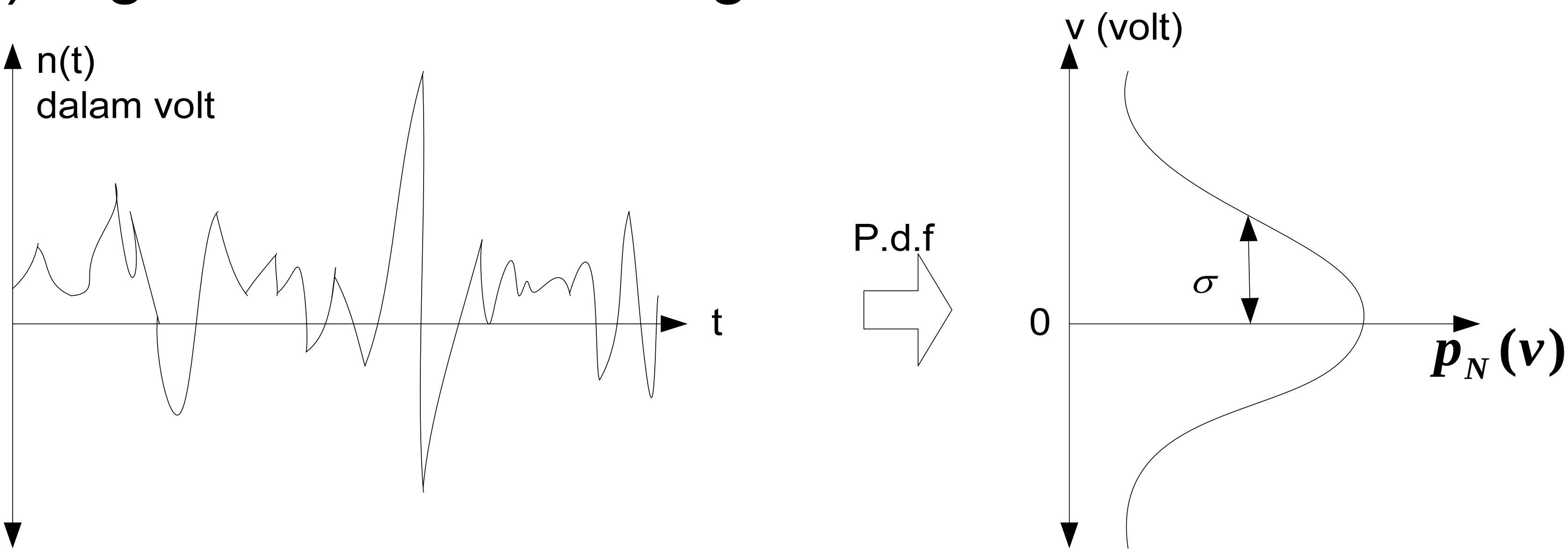
Figure 3-23a Transmitted signal for "10010" using rectangular pulses.

received wave
y(t)=d(t)+n(t)



AWGN

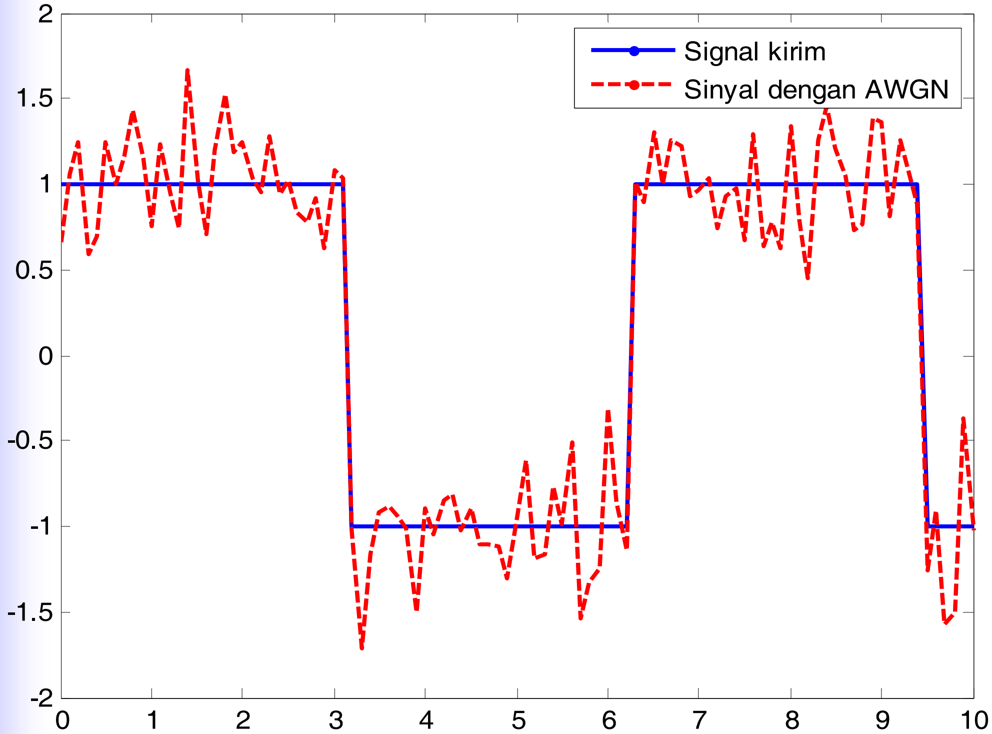
- n(t) = gaussian noise dengan zero-mean

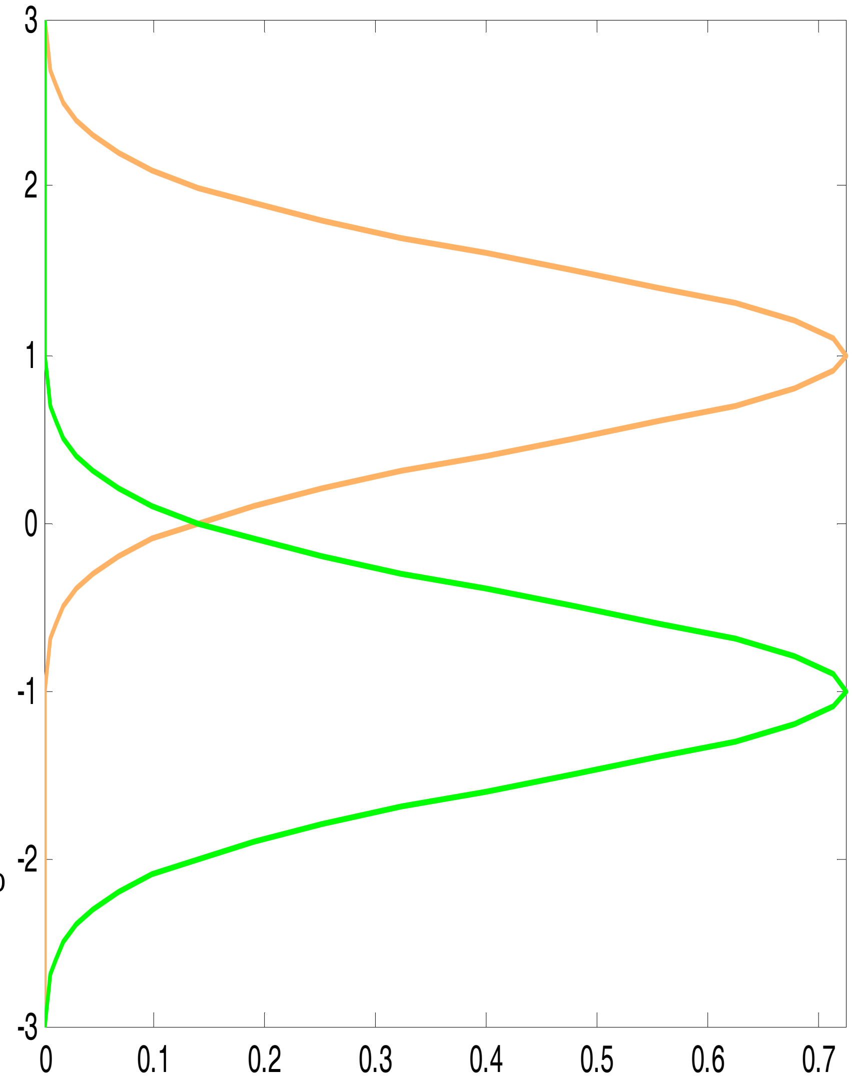


$$p_N(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{v^2}{2\sigma^2}\right]$$

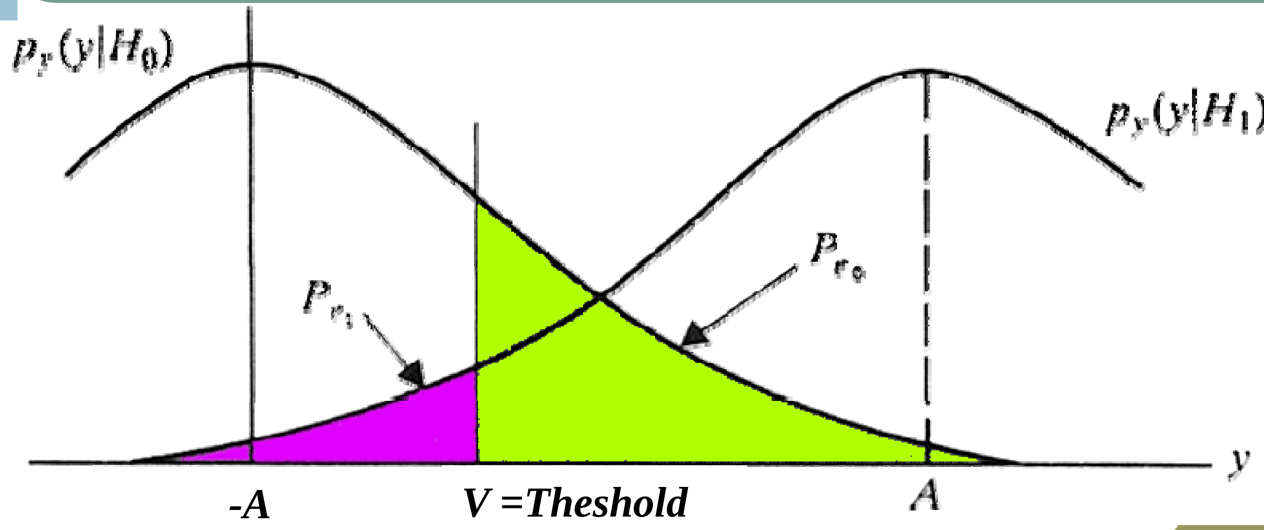
- σ = standar deviasi = tegangan effektive noise

Gangguan Noise Terhadap Sinyal Digital





Determining Decision Threshold



$$H_0 : a_k = 0, Y = -A + n$$
$$p_Y(y / H_0) = p_N(y + A)$$
$$H_1 : a_k = 1, Y = A + n$$
$$p_Y(y | H_1) = p_N(y - A)$$

The comparator implements decision rule:

Choose Ho (a_k=0) if $Y<V$
Choose H1 (a_k=1) if $Y>V$

$$p_{e1} \equiv P(Y < V | H_1) = \int_{-\infty}^{V} p_Y(y | H_1) dy$$
$$p_{eo} \equiv P(Y > V | H_0) = \int_{V}^{\infty} p_Y(y | H_0) dy$$

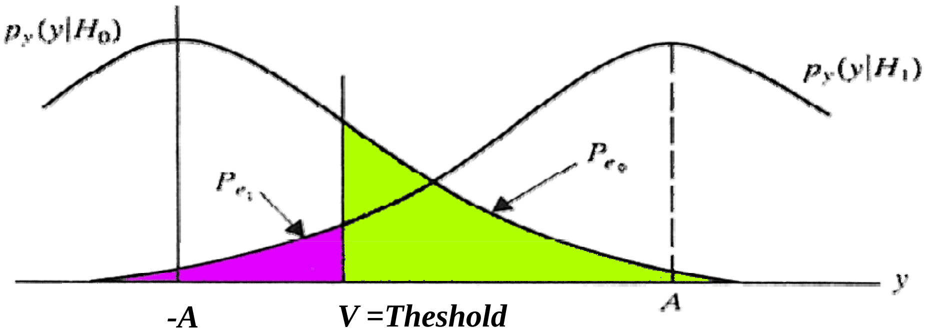
Transmitted ‘0’ but detected as ‘1’

Average error error probability: $P_e = P_0 P_{e0} + P_1 P_{e1}$

$$P_0 = P_1 = 1/2 \Rightarrow P_e = ½(P_{e0} + P_{e1})$$

Channel noise is Gaussian with the pdf: $p_N(y) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left[-\dfrac{y^2}{2\sigma^2}\right]$

Error rate and Q-function



$$P_0 = P_1 = 1/2 \Rightarrow P_e = \tfrac{1}{2}(P_{e0} + P_{e1})$$

V threshold = 0

$$p_{e0} = \int_V^\infty p_N(y)dy$$

$$p_e = p_{e0} = \frac{1}{\sigma\sqrt{2\pi}} \int_{V=0}^{\infty} \exp\left[-\frac{(y+A)^2}{2\sigma^2}\right]dy$$

This can be expressed by using the Q-function

$$Q(z) \triangleq p(x > z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$

by

$$p_{e0} = \int_V^\infty p_N(y)dy = p_e = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{A^2}{\sigma^2}}\right)$$

Baseband Binary Error Rate in Terms of Pulse Shape and S/N

setting $V=0$ yields then

$$p_e = \tfrac{1}{2}(p_{e0} + p_{e1}) = p_{e0} = p_{e1} \Rightarrow p_e = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{A^2}{\sigma^2}}\right) = Q\left(\sqrt{\frac{S}{N}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

for polar, **rectangular** NRZ [-A,A] bits

Probability of occurrence

Signal power: $$S = \frac{1}{2}A^2 + \frac{1}{2}(-A)^2 = A^2$$

Noise power: $$N = \sigma^2 = \eta . BW_N = N_0 . \frac{R_b}{2} = N_0 . \frac{1}{2T_b}$$

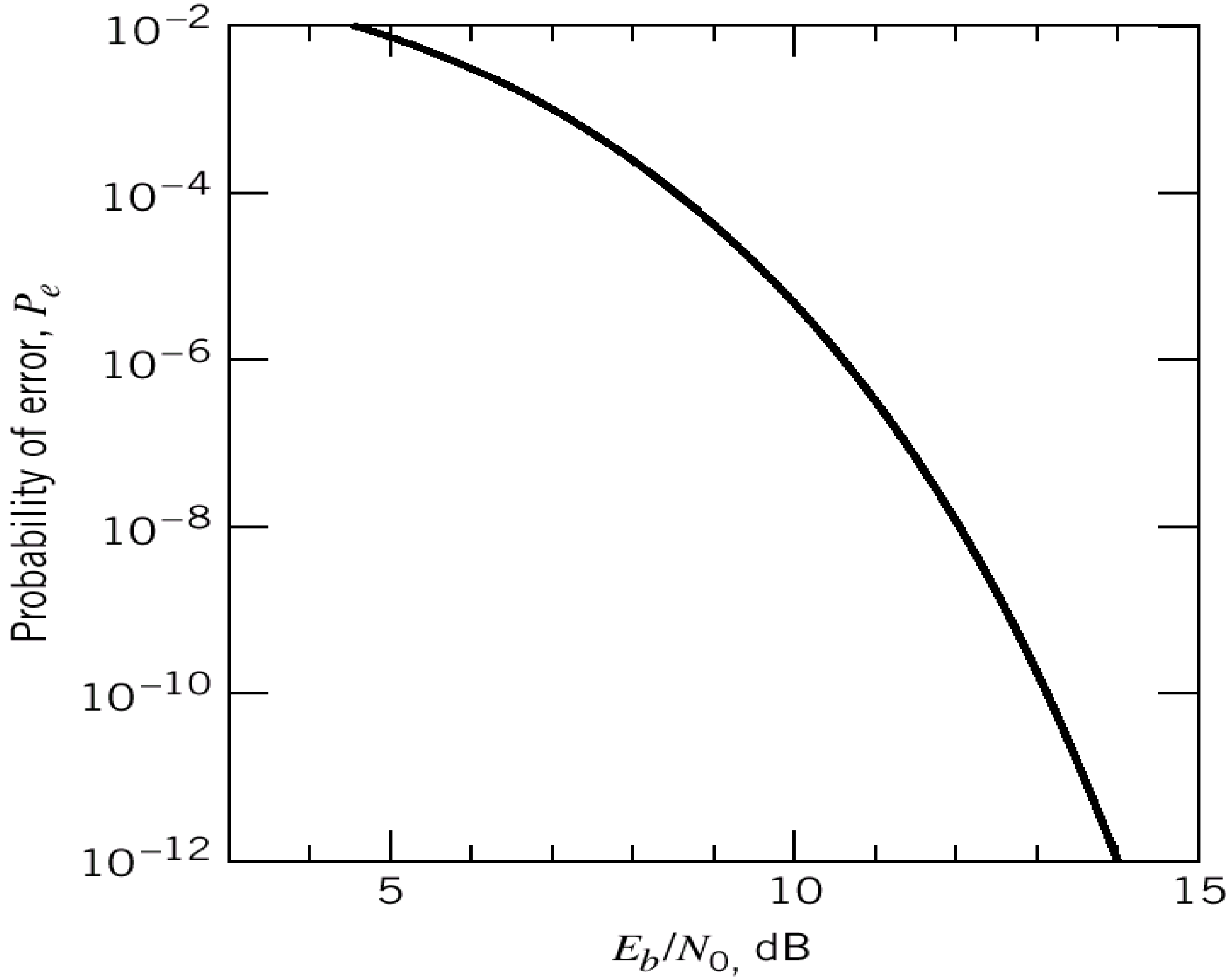
Energy Bit to Noise Spectral Density Ratio

$$\frac{E_b}{N_0} = \frac{S.T_b}{N / BW_N} = \frac{S.T_b}{N / (Rb/2)} = \frac{S.T_b}{N} \cdot \frac{R_b}{2} = \frac{S.T_b}{N} \cdot \frac{1}{2.T_b} = \frac{1}{2} \cdot \frac{S}{N}$$

Note that $BW_N = \dfrac{R_b}{2}$ (BW pulse shapping filter)

When $p_0 = p_1 = 1/2$, the value of V that minimizes the probability of error is V = 0.

$$p_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{S}{N}}\right)$$



Tabel Q-function

TABLE B.1 Complementary Error Function $Q(x) = \int_x^\infty (1/\sqrt{2\pi}) \exp(-u^2/2)\, du$

	$Q(x)$									
x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2168	0.2148
0.8	0.2169	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002