TRANSFORMASI FOURIER

SISTEM KOMUNIKASI
Semester Ganjil 2016/2017
Program Studi S1 Teknik Telekomunikasi
Universitas Telkom

FUNGSI & DEFINISI

- Fungsi Transformasi Fourier yaitu utk menganalisis bentuk spektral [$S(f)$] dari suatu sinyal kawasan waktu [$s(t)$]
- Fungsi Inverse Transformasi Fourier yaitu utk menganalisis bentuk sinyal kawasan waktu [$s(t)$] jika sinyal tersebut memiliki bentuk spektral [$S(f)$]

FORMULA TRANSFORMASI FOURIER

Transformasi fourier

$$S(f) = \int_{-\infty}^{\infty} s(t).e^{-j2\pi ft}\,dt$$

S(f) adalah hasil transformasi fourier dari sinyal dalam domain waktu ***s(t)***

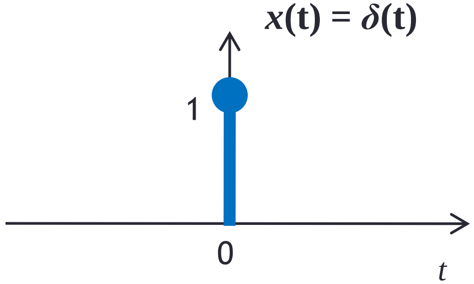
Inverse Transformasi Fourier

$$s(t) = \int_{-\infty}^{\infty} S(f).e^{j2\pi ft}\,df$$

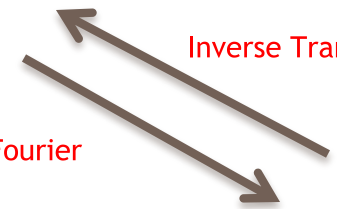
Jika Transformasi Fourier ***S(f)*** suatu sinyal diketahui maka bisa didapatkan kembali persamaan sinyal dalam domain waktu ***s(t)*** dengan formula Inverse Transformasi Fourier

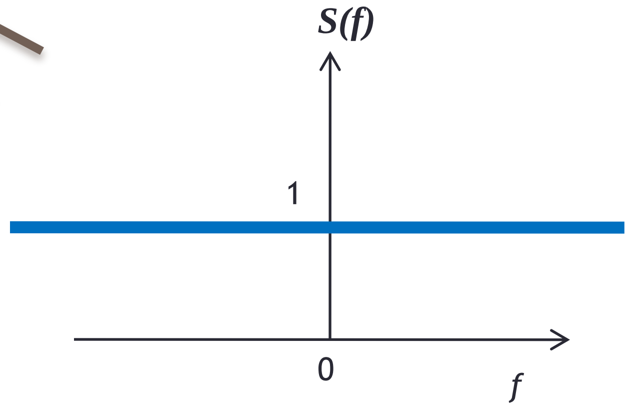
BEBERAPA TRANSFORMASI PENTING

1. Sinyal Delta Diract/ Impuls

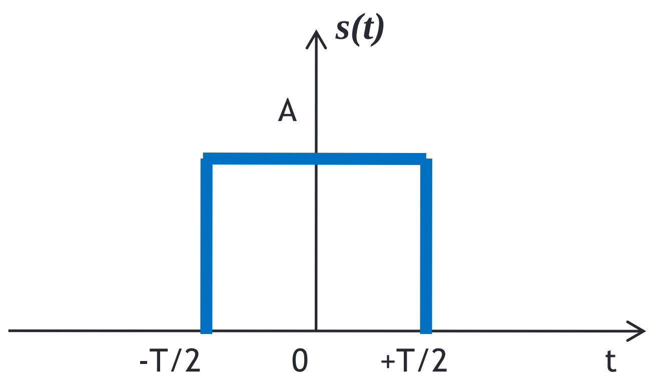


$$S(f) = \int_{-\infty}^{\infty} \delta(t).e^{-j2\pi ft} dt = 1$$

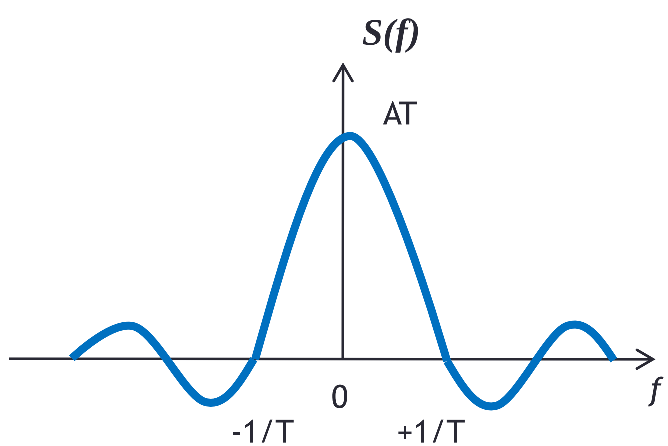




2. Sinyal Rectangular/ Pulsa



Transformasi Fourier ⇅ Inverse Transformasi Fourier



$$S(f) = \int_{-T/2}^{T/2} A\, e^{-j2\pi f t}\, dt$$

$$= \frac{A}{-j2\pi f}\left[e^{-j2\pi f t}\right]_{-T/2}^{T/2}$$

$$= \frac{A}{-j2\pi f}\left[e^{-j2\pi f T/2} - e^{j2\pi f T/2}\right]$$

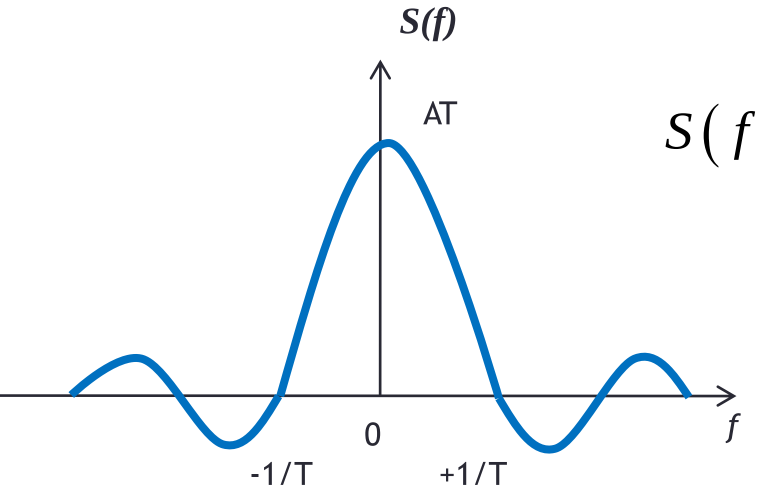
$$= \frac{A}{j2\pi f}\left[e^{j2\pi f T/2} - e^{-j2\pi f T/2}\right]$$

$$= \frac{A}{\pi f}\,\frac{1}{2j}\left[e^{j2\pi f T/2} - e^{-j2\pi f T/2}\right]$$

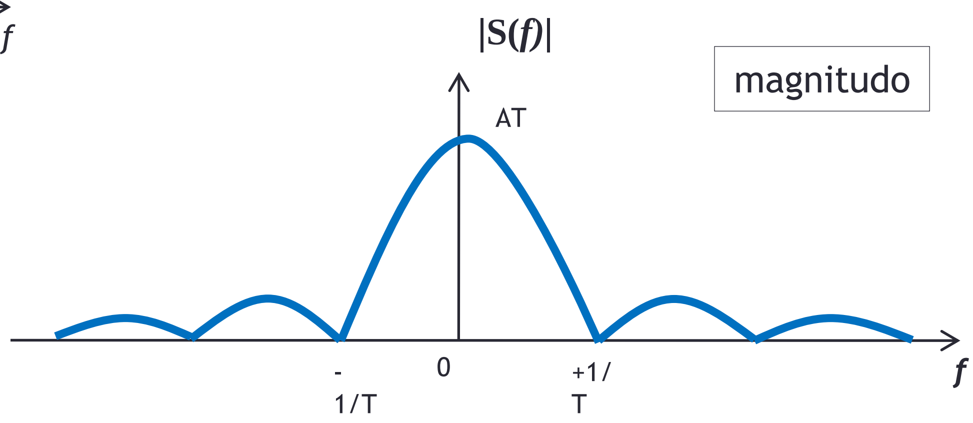
$$= \frac{A}{\pi f}\sin \pi f T$$

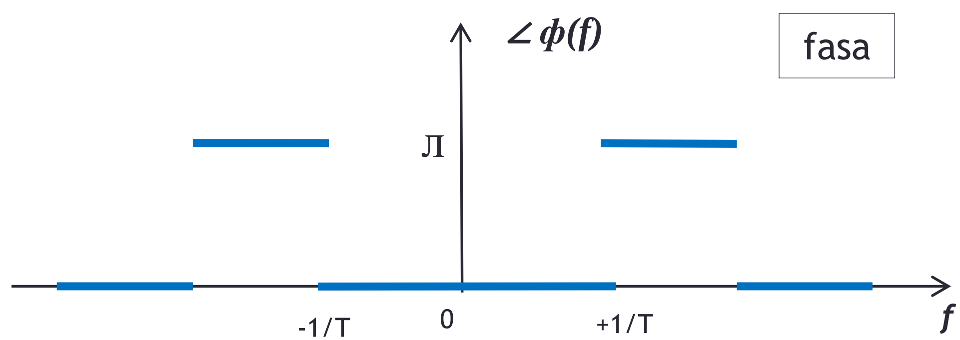
$$= \frac{A}{1/T}\,\frac{\sin(\pi f T)}{\pi f T}$$

$$= AT.\operatorname{sinc}(\pi f T)$$



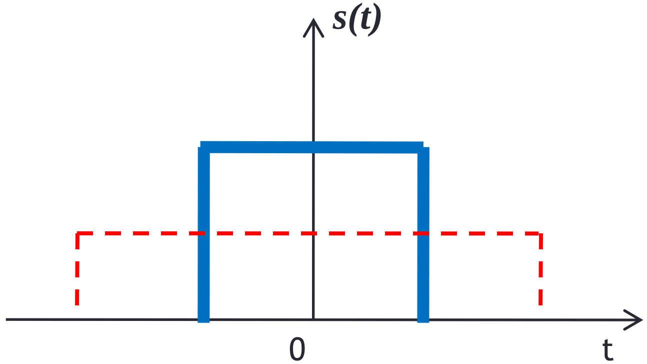
$$S(f) = AT.\sin c(\pi f T)$$





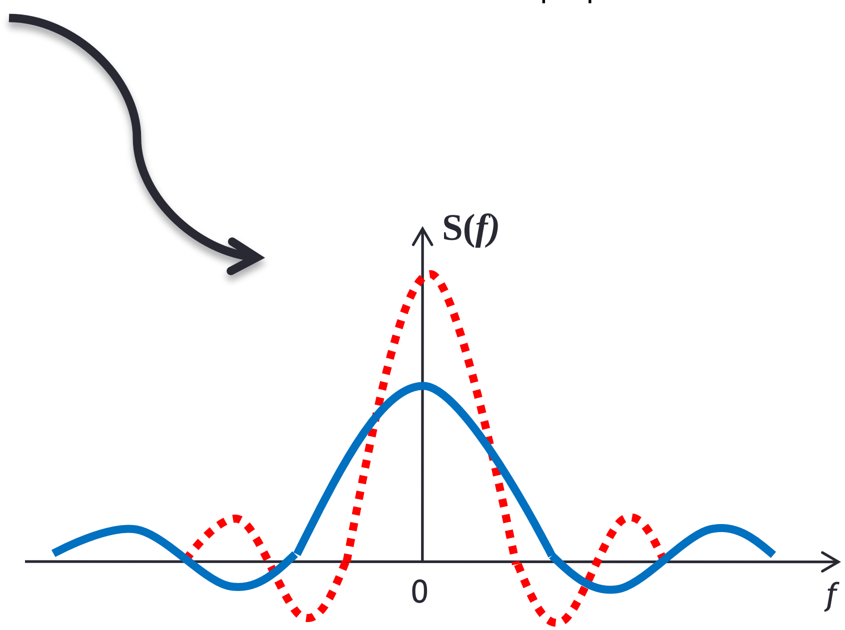
SIFAT-SIFAT TRANSFORMASI FOURIER

a. Time Scaling



Jika: $s(t) \Leftrightarrow S(f)$

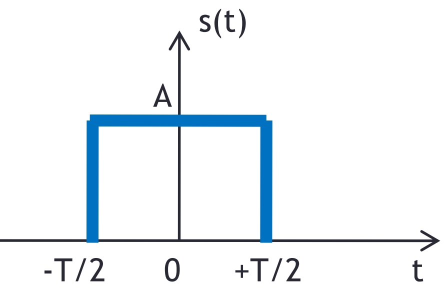
Maka: $s(at) \Leftrightarrow \frac{1}{|a|}.S\left(\frac{f}{a}\right)$

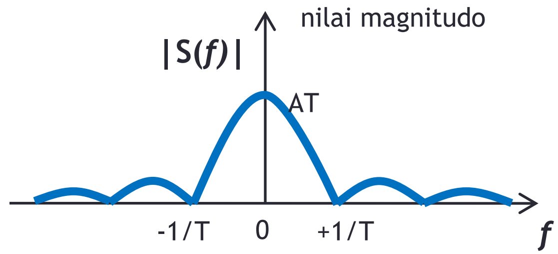


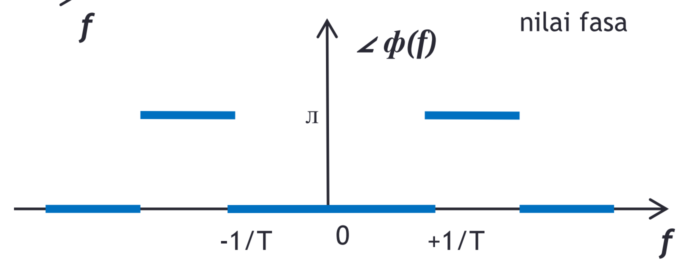
b. Time Shift

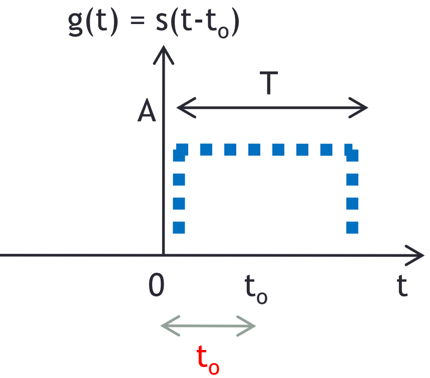
Jika $s(t) \longleftrightarrow S(f)$

maka $s(t-t_o) \longleftrightarrow S(f)\, e^{-j2.\pi f. t_o}$

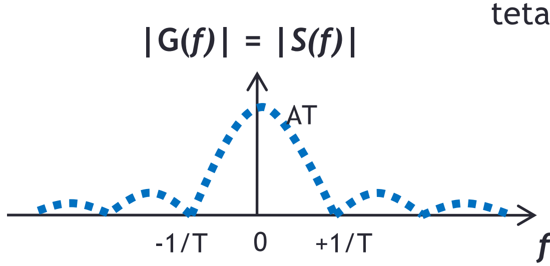




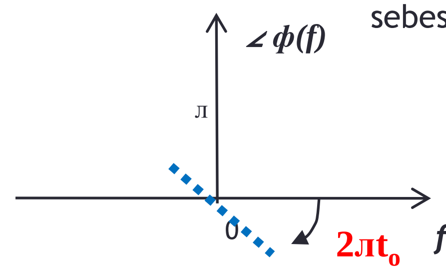




nilai magnitudo tetap



nilai fasa ada pergeseran sebesar $2\pi t_o$

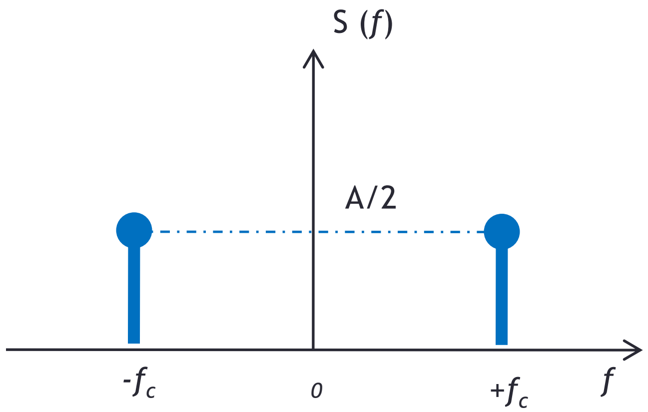


c. Frequency Shift

Jika s(t) ⟷ S(f) maka $S(f-f_o) \longleftrightarrow s(t)\, e^{-j2\pi . f_o . t}$

Contoh: $s(t) = A.\cos 2\pi f_c t = \frac{A}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$

maka $S(f) = \frac{A}{2}\delta(f + f_c) + \frac{A}{2}\delta(f - f_c)$



d. Transformasi Fourier Sinyal Periodik

Jika $x(t) \longleftrightarrow X(f)$ untuk sinyal non-periodik,

Maka

$$x_p(t) = \sum_{n-\infty}^{+\infty} x(t - nT_o)$$

$x_p(t)$ sinyal periodik dengan periode T_o

Transformasi Fourier ↓ ↑ Inverse Transformasi Fourier

$$X_p(f) = \frac{1}{T_o} \sum_{m=-\infty}^{+\infty} X\left(\frac{m}{T_o}\right) . \delta\left(f - \frac{m}{T_o}\right)$$

e. Integrasi pada kawasan waktu `

Bila $s(t) \longleftrightarrow S(f)$, kemudian menghasilkan $S(0) = 0$, maka

$$\int_{-\infty}^{t} s(t).\,dt \leftrightarrow \frac{1}{j2\pi f}\,.S(f)$$

f. Diferensiasi pada kawasan waktu

Bila $s(t) \longleftrightarrow S(f)$, Jika pada kawasan waktu dilakukan diferensiasi sekali maka:

$$\frac{d}{dt}s(t) \leftrightarrow j2\pi f.S(f)$$

g. Konvolusi pada kawasan waktu

Jika $s_1(t) \longleftrightarrow S_1(f)$ dan $s_2(t) \longleftrightarrow S_2(f)$, maka

$$\int_{-\infty}^{\infty} s_1(t).s_2(t-\tau)d\tau \leftrightarrow S_1(f).S_2(f)$$

h. Perkalian pada kawasan waktu

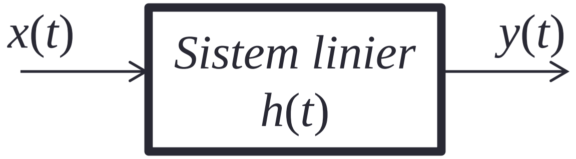
Jika $s_1(t) \longleftrightarrow S_1(f)$ dan $s_2(t) \longleftrightarrow S_2(f)$, maka

$$s_1(t).s_2(t) \leftrightarrow \int_{-\infty}^{\infty} S_1(\lambda).S_2(f-\lambda)d\lambda$$

TRANSMISI SINYAL MELALUI SISTEM LINIER

Respon waktu:

time domain



$h(t)$ Ξ respon impuls

$$y(t) = \int_{-\infty}^{\infty} h(\lambda).x(t-\lambda).d\lambda$$

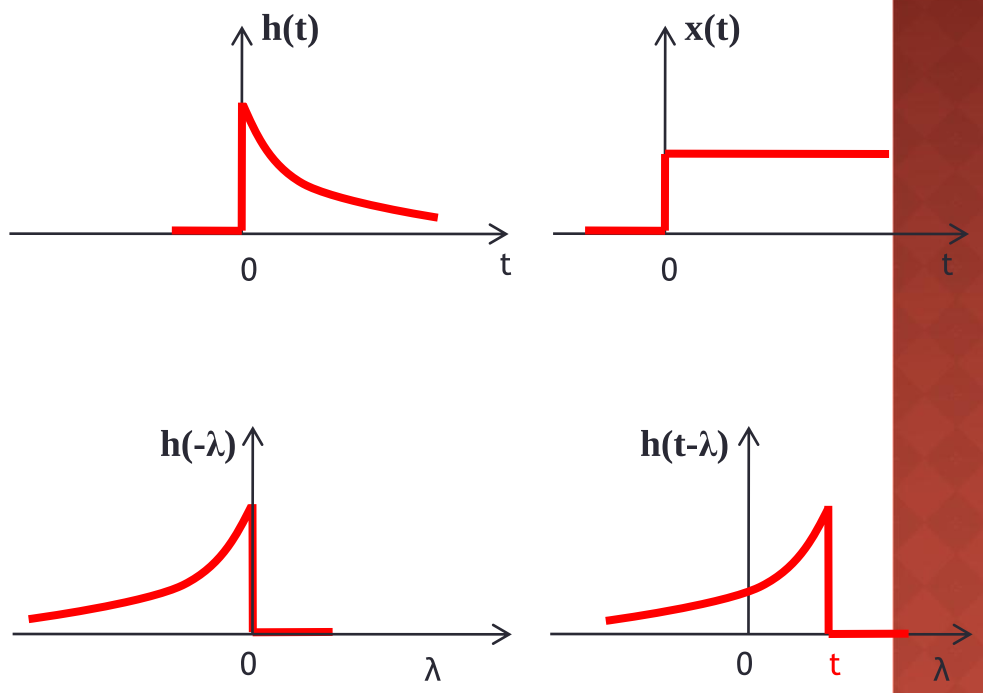
$$= \int_{-\infty}^{\infty} x(\lambda).h(t-\lambda).d\lambda$$

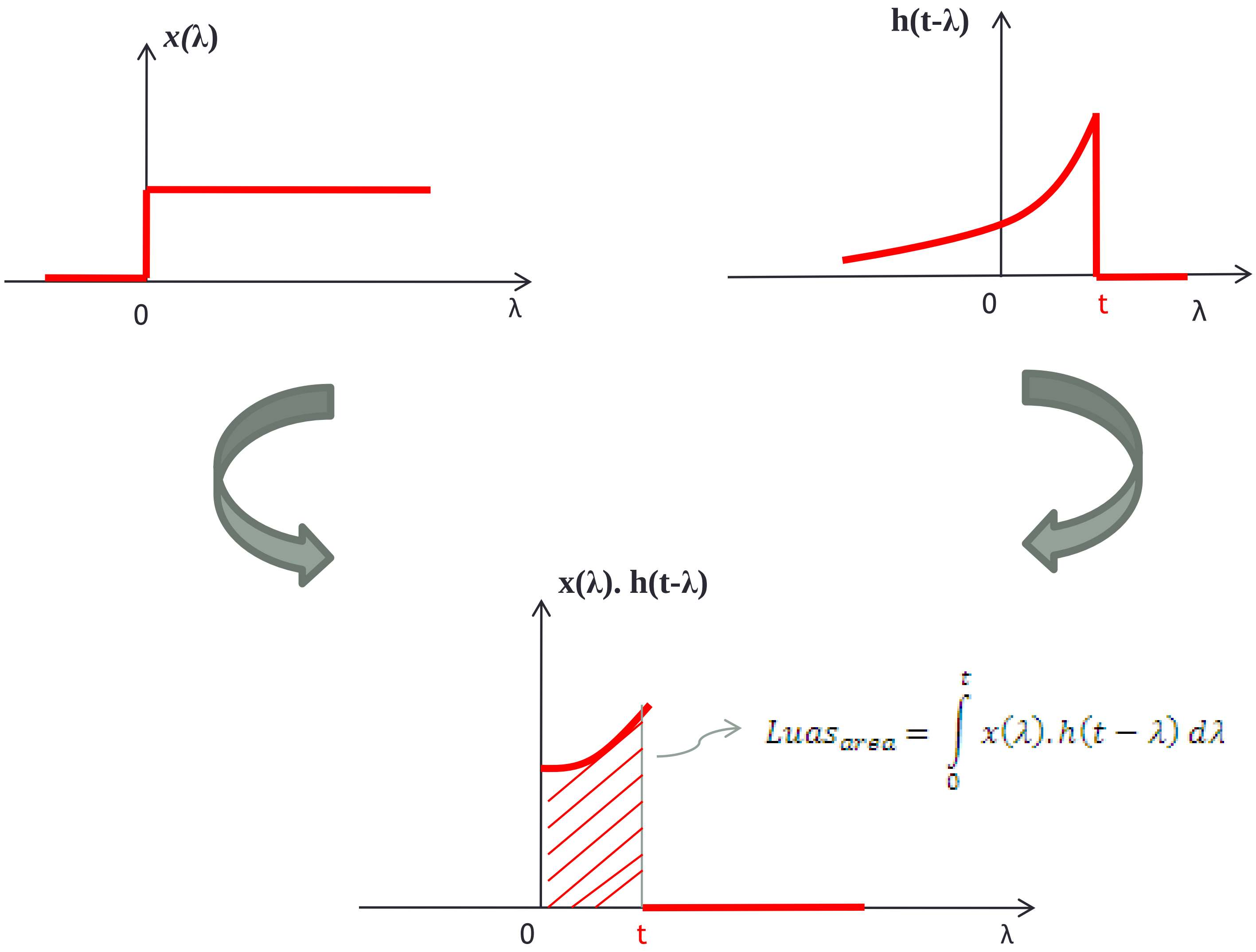
$$= x(t) \otimes h(t)$$

$$= h(t) \otimes x(t)$$

Contoh: perhitungan konvolusi, representasi grafis

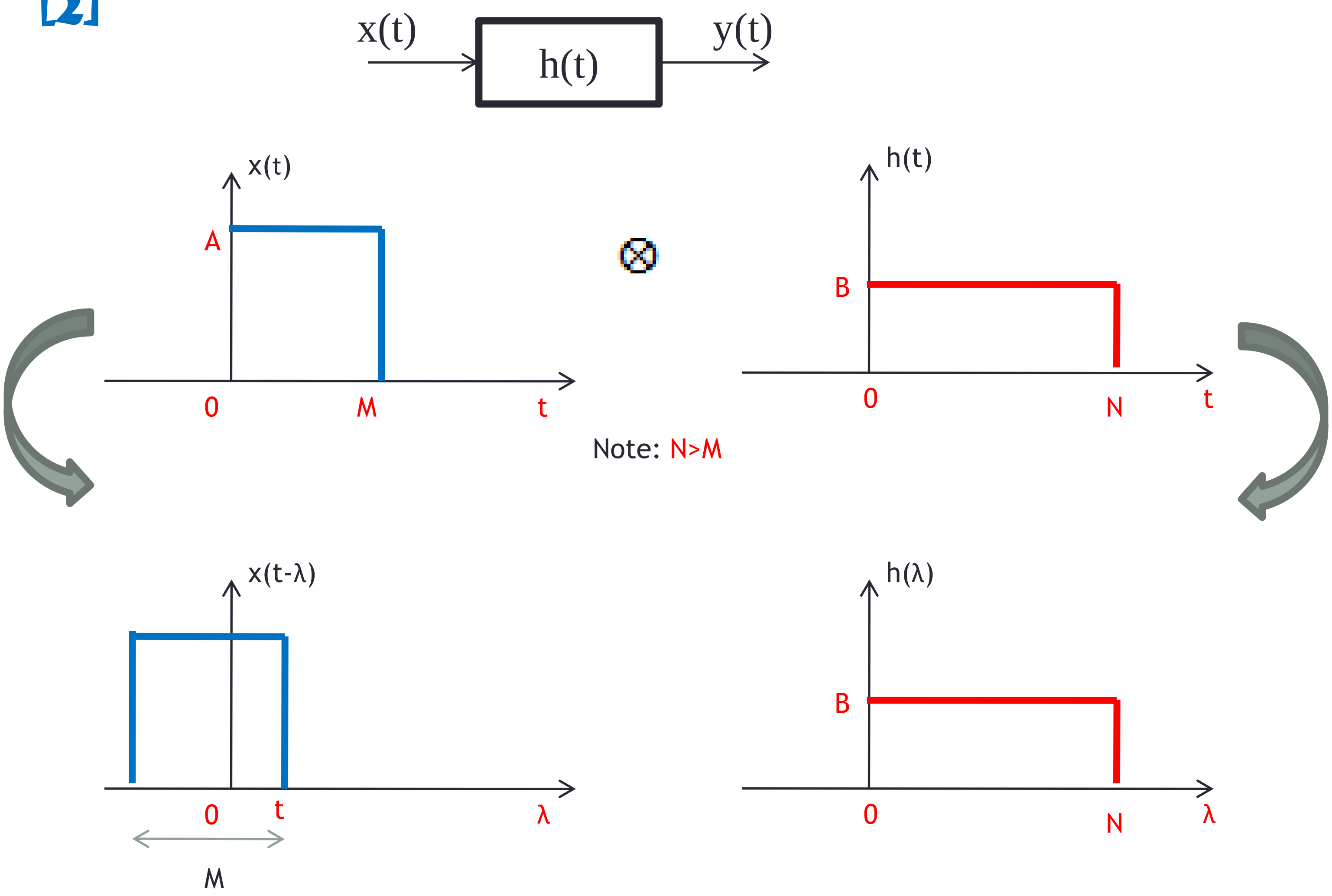
[1]



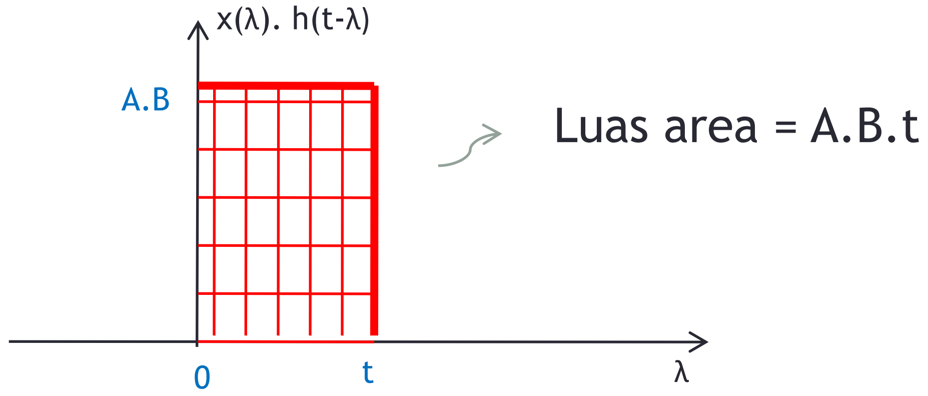


$$Luas_{area} = \int_0^t x(\lambda).h(t-\lambda)\, d\lambda$$

[2]

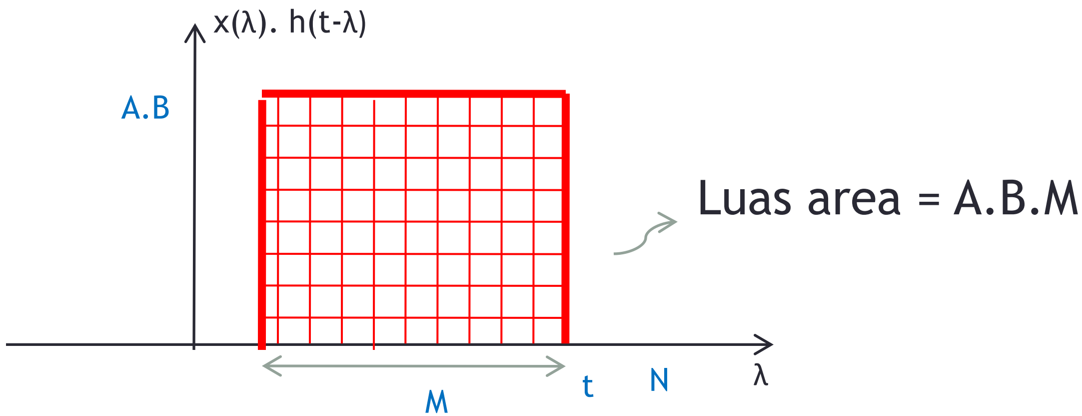


Untuk $0 \leq t \leq M$, maka:

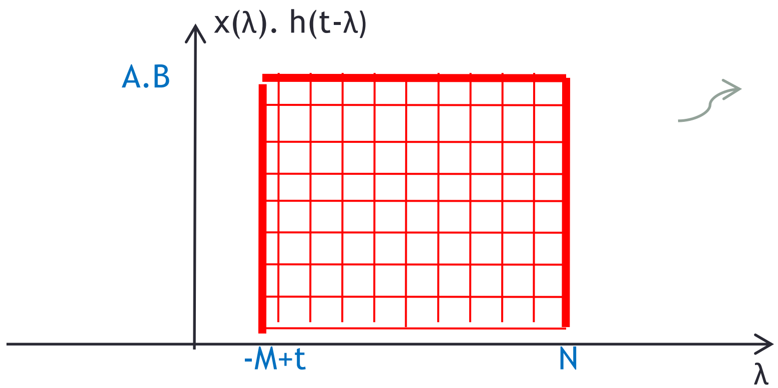


Untuk $M < t \leq N$, maka:

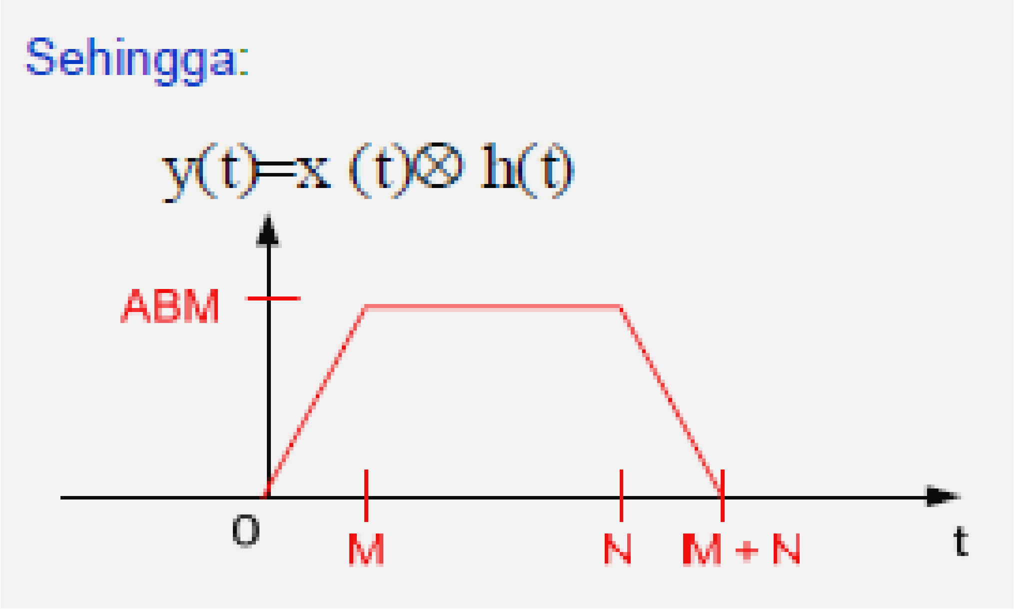
M



Untuk t ≥ N, maka:



Luas area = A.B. (N+M-t)

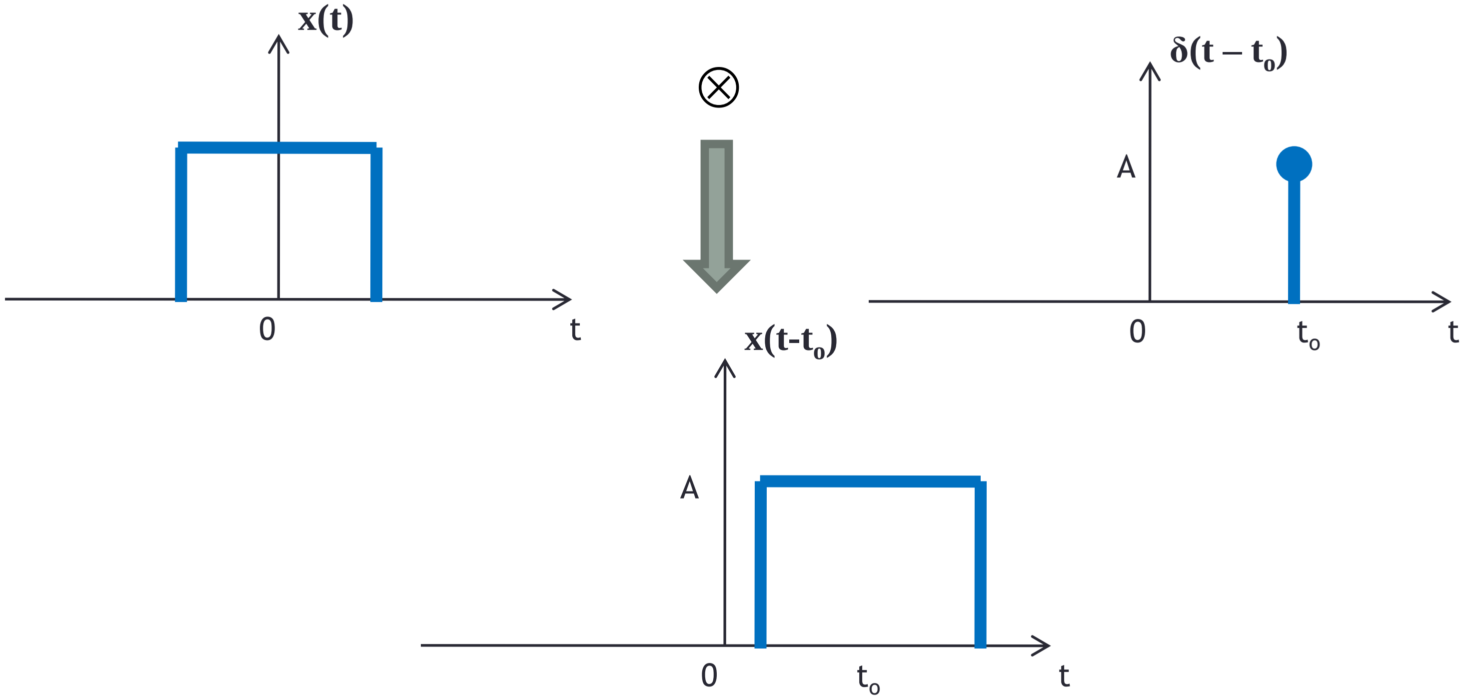


KASUS KHUSUS

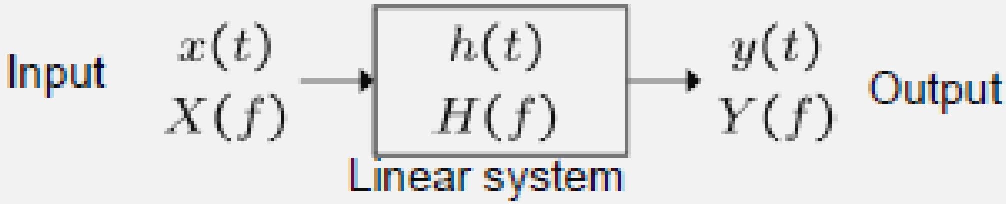
Konvolusi dengan fungsi ***δ (t-t$_o$)***

$$x(t)\otimes\delta(t-t_o)=\int_{-\infty}^{\infty}x(t-\lambda).\delta(t-t_o)d\lambda=x(t-t_o)$$

$$x(t)\otimes A.\delta(t-t_o)=A.x(t-t_o)$$



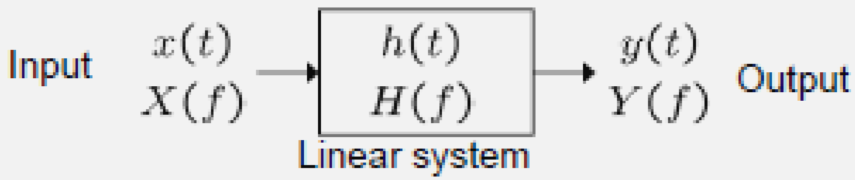
TRANSMISI SINYAL MELALUI SISTEM LINIER



- Deterministic signals: $Y(f) = X(f)H(f)$
- Random signals: $G_Y(f) = G_X(f)|H(f)|^2$

- $Y(f)$ = Sinyal output dalam domain frekuensi
- $X(f)$ = Sinyal input dalam domain frekuensi
- $H(f)$ = Respons frekuensi sistem linier
- $G_Y(f)$ = PSD (Power Spectral Density) sinyal output
- $G_X(f)$ = PSD (Power Spectral Density) sinyal input

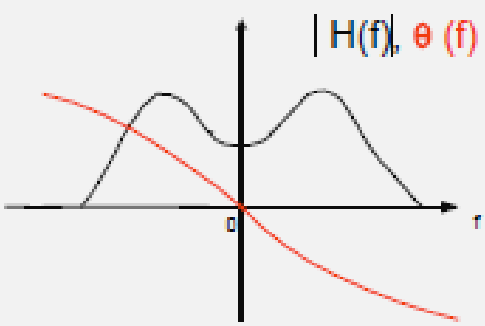
Sistem Lowpass vs Bandpass



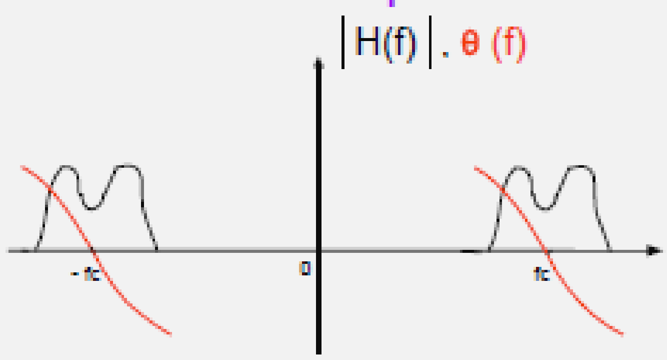
Jika h (t) riil ⇒ H (f) kompleks → | H (f) | merupakan fungsi genap

→ θ (f) merupakan fungsi ganjil

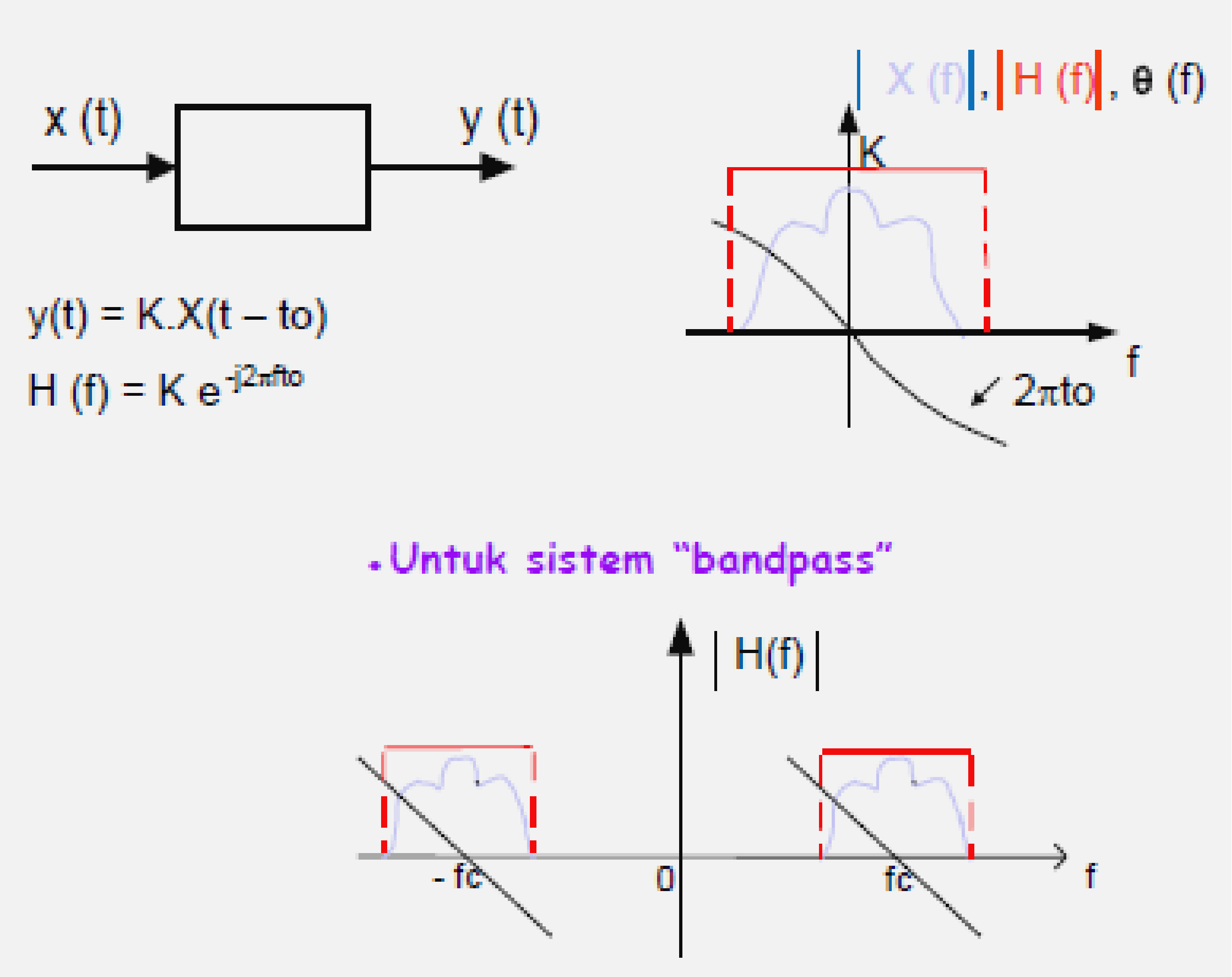
Sistem "lowpass"



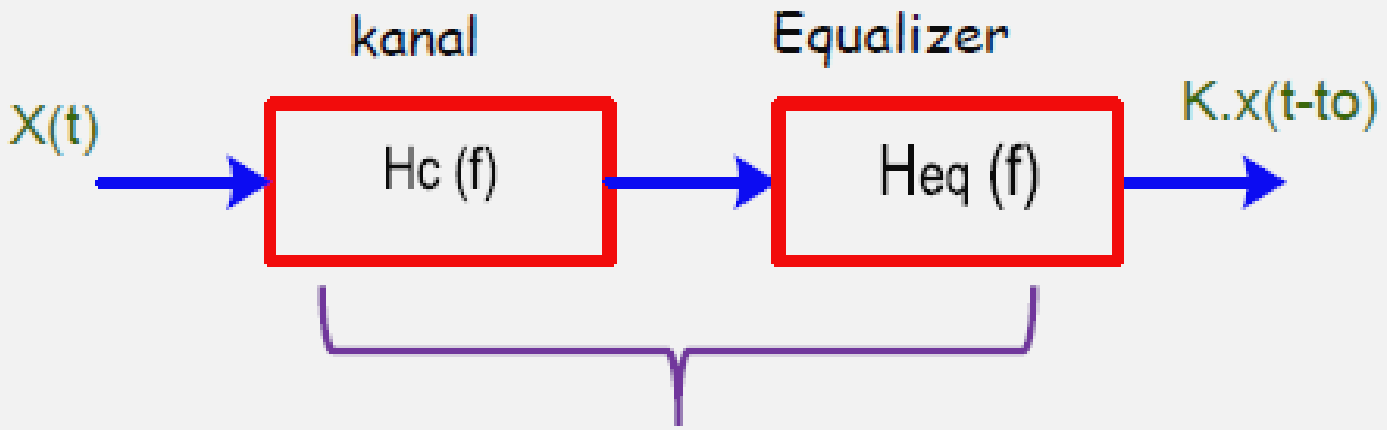
Sistem "bandpass"



Kondisi "distortionless transmission"



Kondisi "distorsi linier" dan Prinsip Ekualisasi Kanal

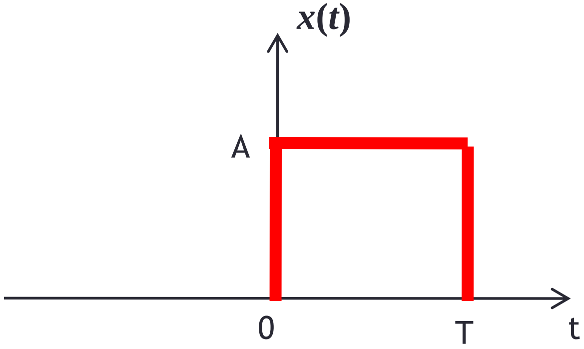


$$Hc(f)\,Heq(f) = K\,e^{-j2\pi f t_o}$$

$$Heq(f) = \frac{K\,e^{-j2\pi f t_o}}{Hc(f)}$$

LATIHAN SOAL

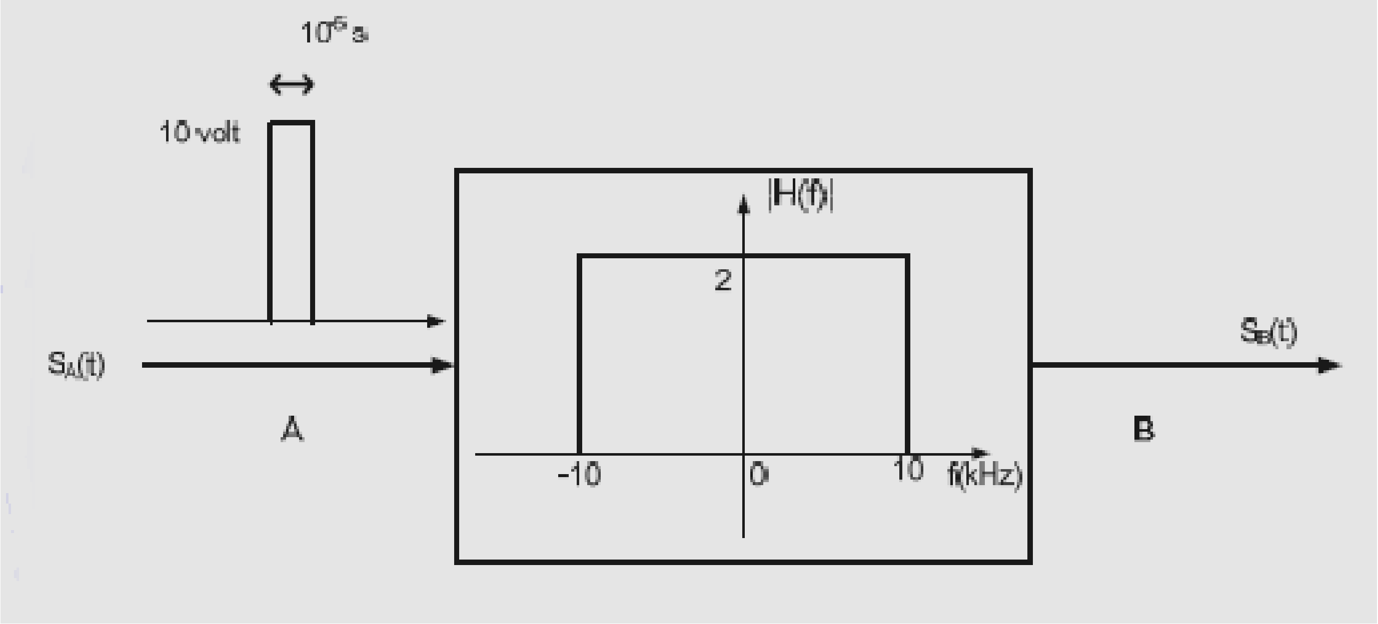
[1] Perhatikan gambar sinyal $x(t)$ ***dibawah ini:***



a. Tentukan $X(f)$ yang merupakan transformasi fourier dari sinyal tersebut !

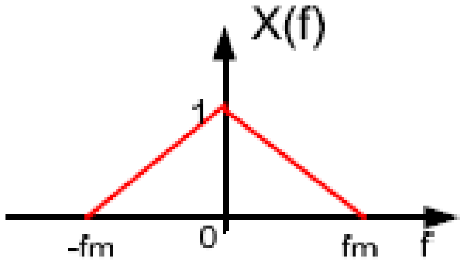
b. Jika sinyal $z(t) = x(t) * y(t)$, dimana $y(t) = \cos(4\pi t/T)$, tentukan $Z(f)$

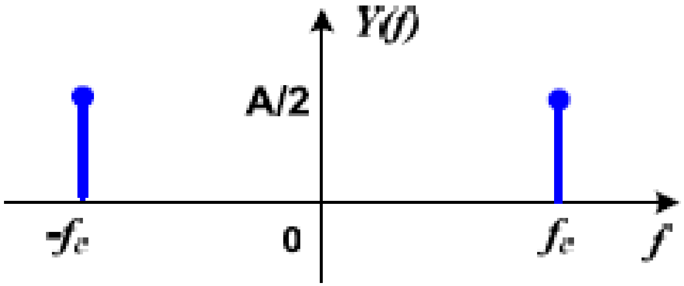
[2] Suatu sinyal memasuki sistem yang diwakili oleh LPF berikut ini:



Tentukan $S_A(f)$, $S_B(f)$, $S_B(t)$!

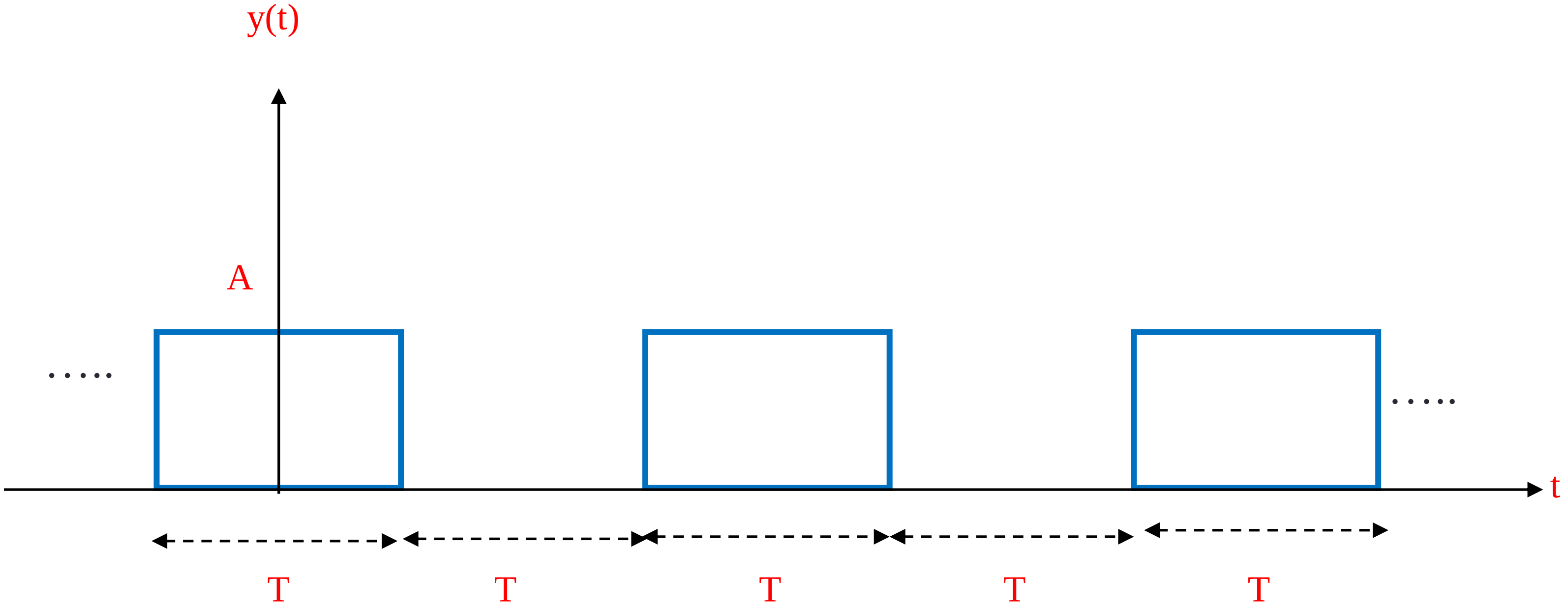
[3] Diketahui sinyal dalam domain frekuensi sebagai berikut:



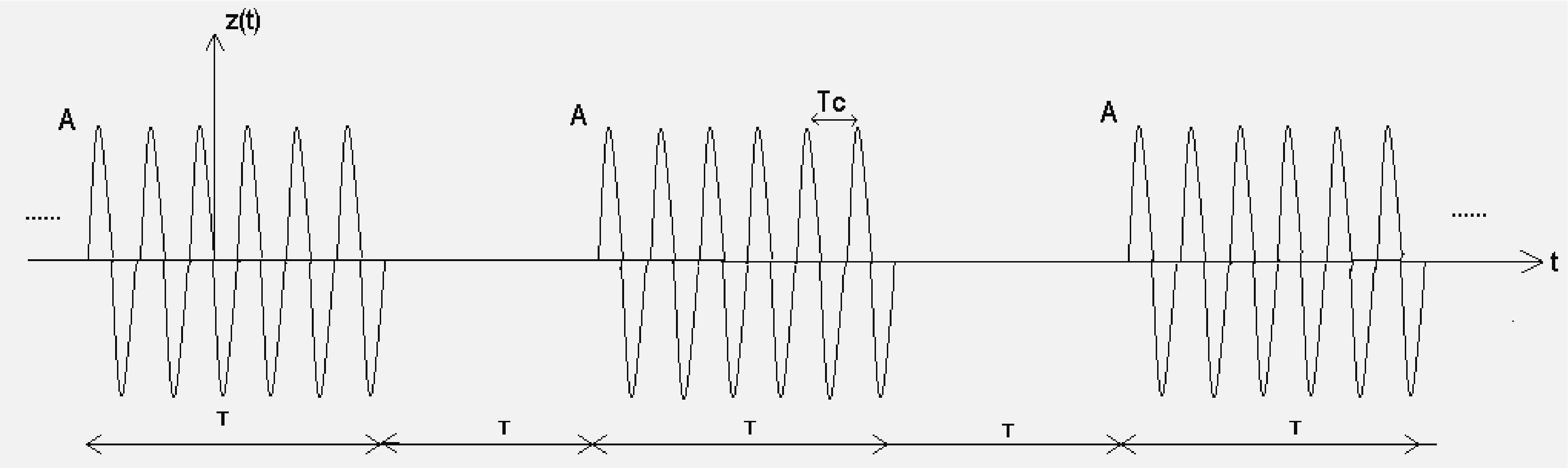


Untuk $f_c > f_m$, Gambarkan $Z(f) = X(f) * Y(f)$!

[4] Tentukanlah *Y(f)* dan gambarkan jika diketahui gambar *y(t)* berikut ini!



[5] Tentukanlah *Z(f)* dan gambarkan jika diketahui gambar *z(t)* berikut ini!



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