Sistem Komunikasi II
(*Digital Communication Systems*)

Lecture #3: **Demodulasi / Deteksi Baseband**
(*Baseband Demodulation / Detection*)
- PART I –

Topik:

3.1 Pendahuluan.

3.2 Representasi Geometris dari Sinyal.

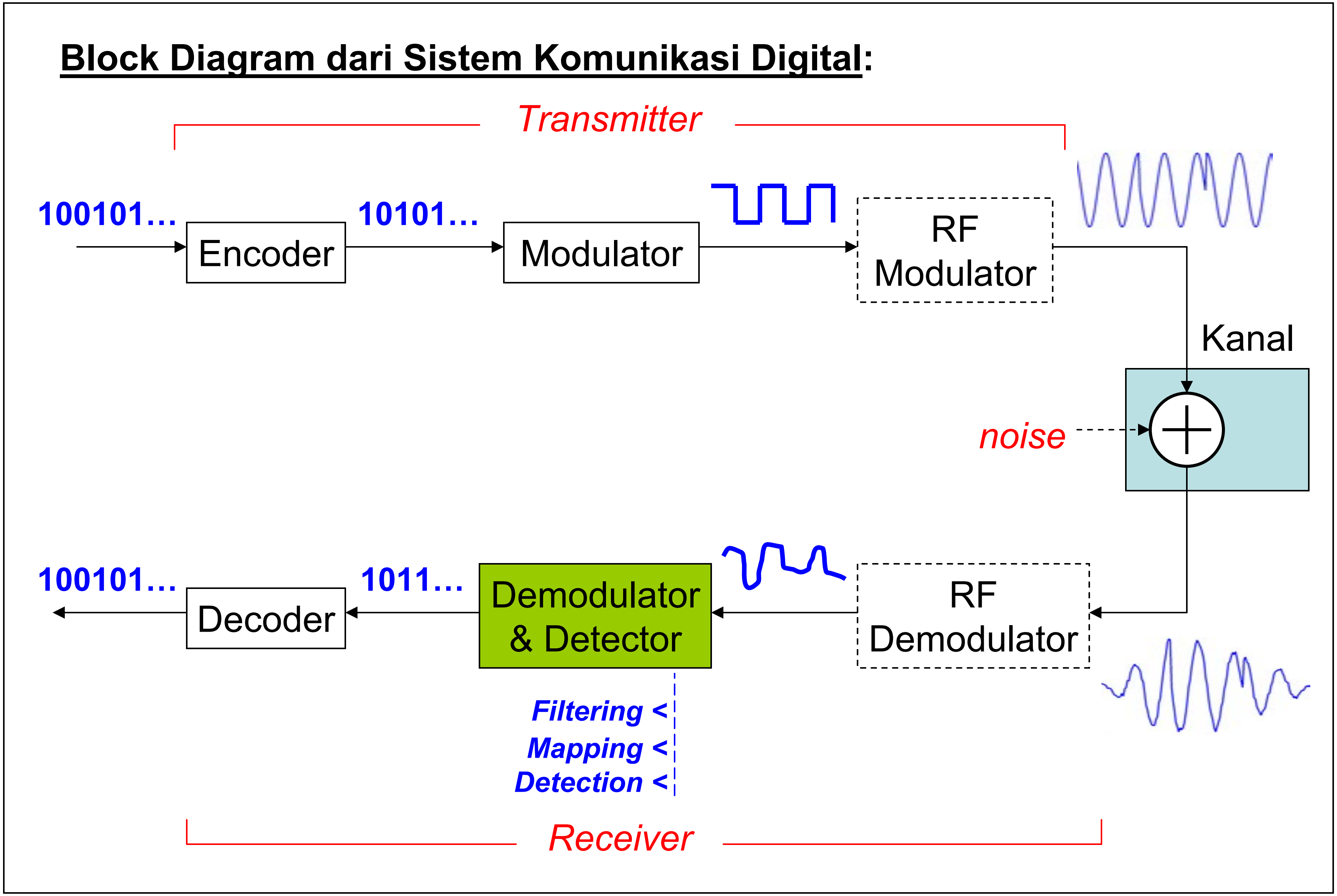
3.3 Optimal Detection: "*Maximum Likelihood Detection*".

3.4 Energy/Symbol, Energy/Bit, and Minimum Distance.

3.5 Probabilitas Error untuk Transmisi Binary PAM dengan (Optimal) Maximum Lkelihood Detection.

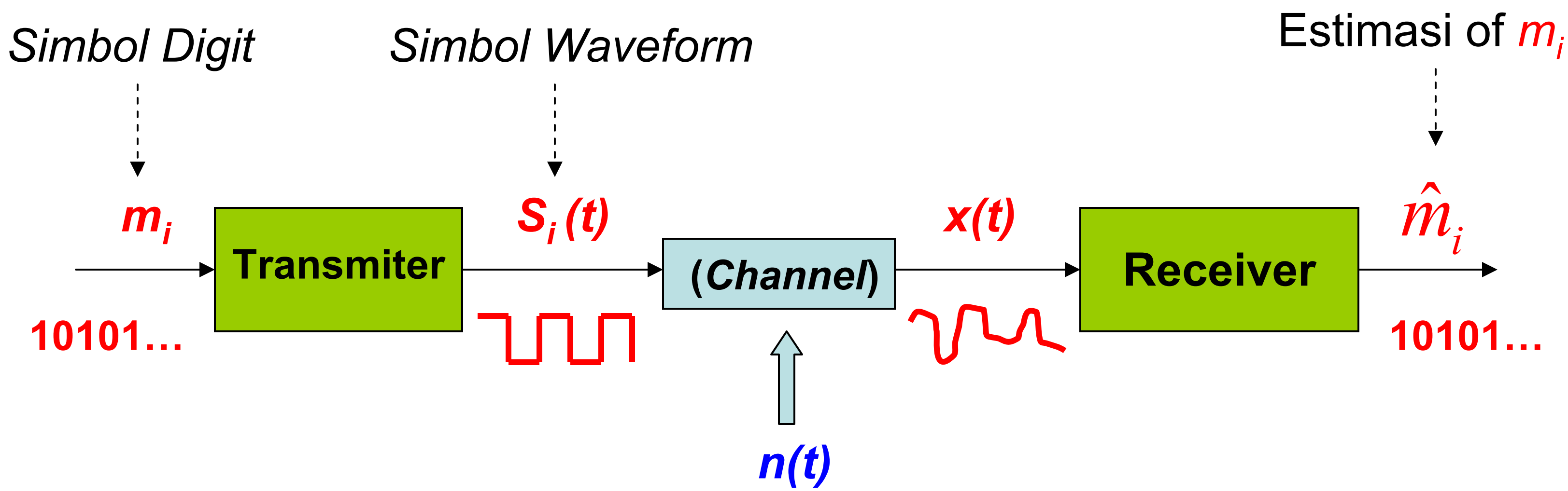
3.6 Optimal Filter: "Matched Filter" or "Correlator

3.1. Introduction



3.1. Pendahuluan – cont.

Sistem Komunikasi Digital (*Baseband*):



m_i *adalah simbol digit yang me-representasikan informasi digital (message).*

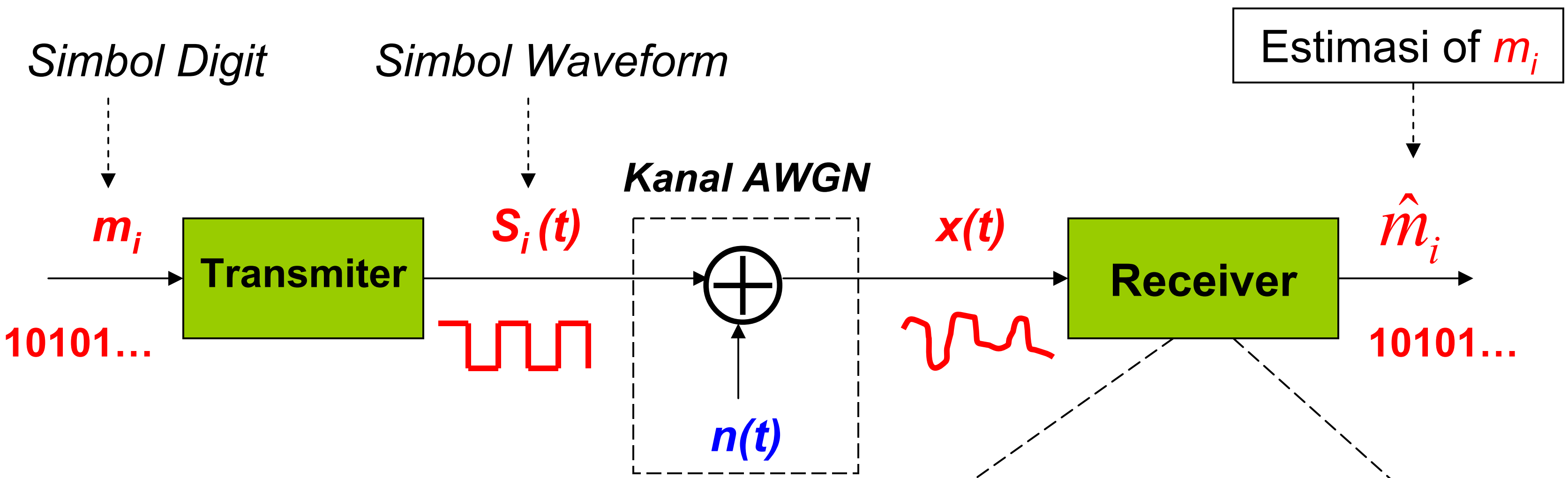
$$m_i \in [m_1, m_2, \ldots, m_M] \quad \leftarrow \textit{Alfabet simbol}$$

Contoh:

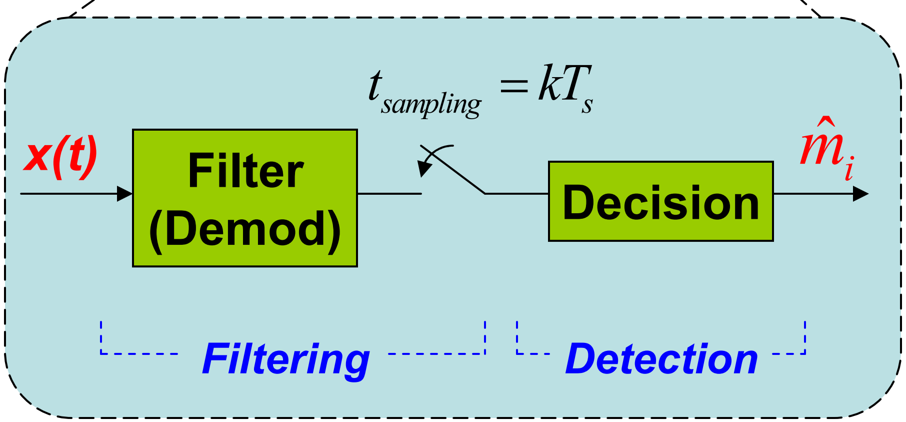
1. Binary PAM: $m_1 = 0$, $m_2 = 1$
2. 4-ary PAM: $m_1 = 00$, $m_2 = 01$, $m_3 = 10$, $m_4 = 11$

3.1. Pendahuluan – cont.

Sistem Komunikasi Digital (*Baseband*):



$n(t)$ is White Gaussian Noise (WGN).



Goals:

1. ***Menentukan bentuk filtering yang optimal.***
2. ***Menentukan bentuk detection yang optimal.***

3.2. Representasi Geometris dari Sinyal

Representasi Geometris dari sinyal $s_i(t)$:

Sintesis

Ekspansi

$$s_i(t) = \sum_{j=1}^{N} s_{ij} \cdot \phi_j(t) \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases}$$

$$\boxed{N \le M}$$

Koefisien Ekspansi

$$s_{ij} = \int_0^T s_i(t) \cdot \phi_j(t)\, dt \qquad \begin{cases} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{cases}$$

Analisis

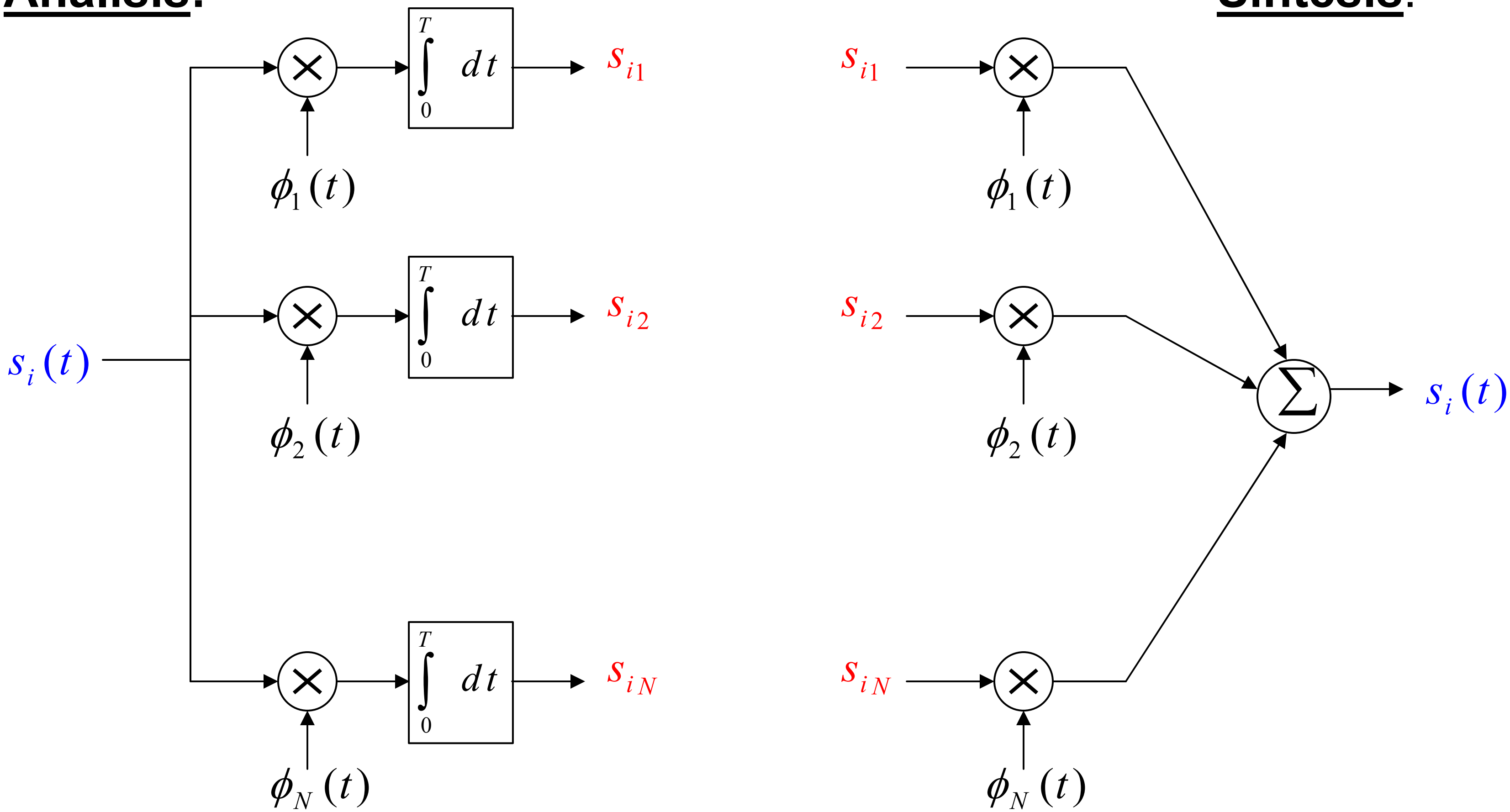
Fungsi Basis Orthonormal

$$\phi_j(t) \quad ; j = 1, 2, \ldots, N$$

$$Orthonormal : \quad \int_0^T \phi_i(t) \cdot \phi_j(t)\, dt = \begin{cases} 1 & ; i = j \\ 0 & ; i \ne j \end{cases}$$

3.2. Representasi Geometris dari Sinyal – cont.

Analisis: **Sintesis:**

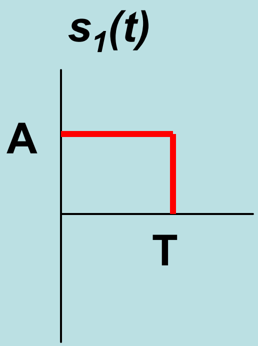


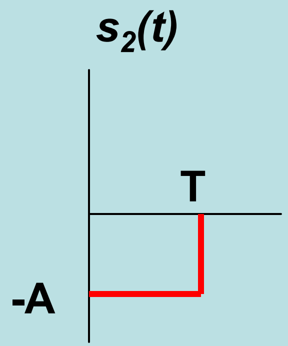
$$s_{ij} = \int_0^T s_i(t) \cdot \phi_j(t)\, dt \qquad ; j = 1, 2, \ldots, N$$

$$s_i(t) = \sum_{j=1}^{N} s_{ij} \cdot \phi_j(t) \qquad ; 0 \le t \le T$$

3.2. Representasi Geometris dari Sinyal – cont.

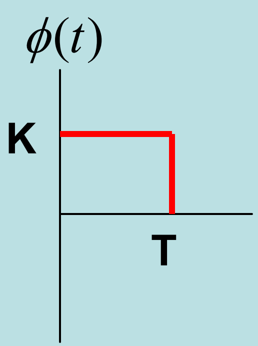
Contoh: Binary PAM (NRZ)





$m_1 = 1$ *$m2 = 0$*

Secara intuitif … fungsi basis:



Tapi, $K = ?$

Ingat …

$$\int_0^T \phi_i(t) \cdot \phi_j(t)\, dt = \begin{cases} 1 & ; i = j \\ 0 & ; i \neq j \end{cases}$$

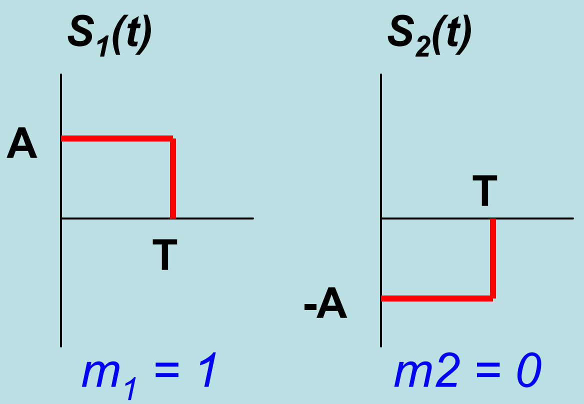
$$\int_0^T \phi_i^2(t)\, dt = 1$$

$$\int_0^T \phi^2(t)\, dt = K^2 T$$

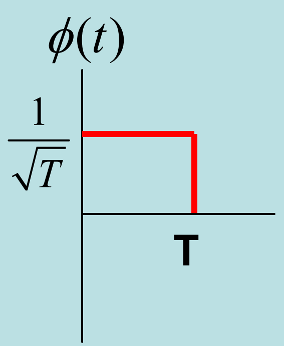
$$K = \frac{1}{\sqrt{T}}$$

3.2. Representasi Geometris dari Sinyal – cont.

Contoh: Binary PAM – cont.



Fungsi Basis:



Analisis (koefisien ekspansi):

$$s_1 = \int_0^T s_1(t) \cdot \phi(t)\, dt = A\sqrt{T}$$

$$s_2 = \int_0^T s_2(t) \cdot \phi(t)\, dt = -A\sqrt{T}$$

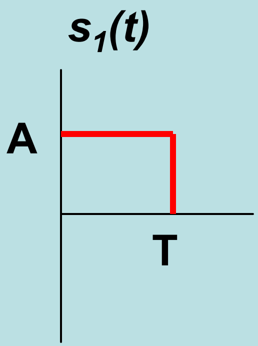
Sintesis:

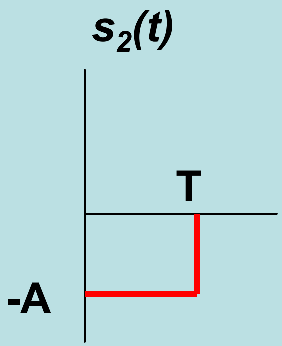
$$s_1(t) = s_1 \cdot \phi(t) = A\sqrt{T} \cdot \phi(t)$$

$$s_2(t) = s_2 \cdot \phi(t) = -A\sqrt{T} \cdot \phi(t)$$

3.2. Representasi Geometris dari Sinyal – cont.

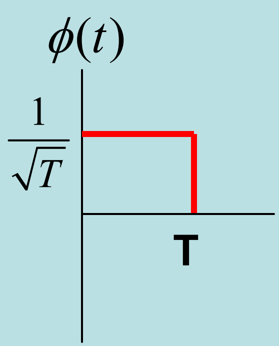
Contoh: Binary PAM – cont.





m_1 = 1 *m2 = 0*

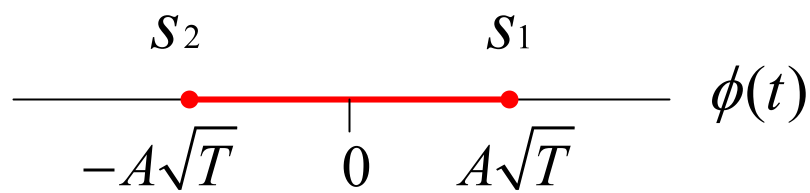
Fungsi Basis:



$$S_1 = A\sqrt{T}$$

$$S_2 = -A\sqrt{T}$$

Representasi Geometris:

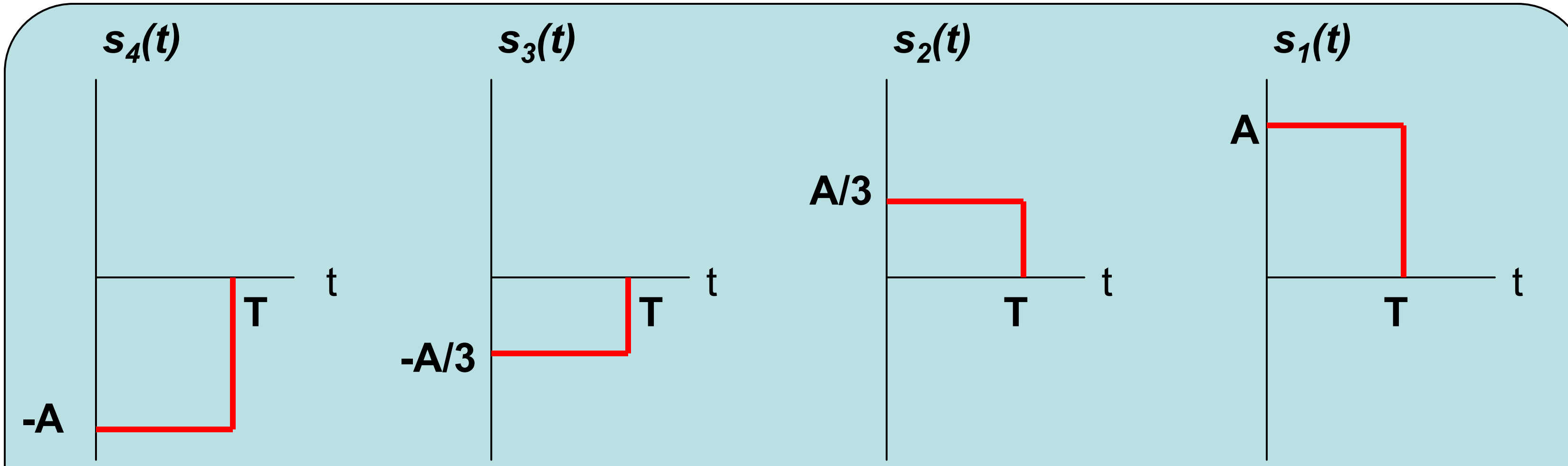


Signal Space (Konstelasi Sinyal)
1-Dimension (1D)

- 1 fungsi basis
- $s_i(t)$ → s_i ~ sample

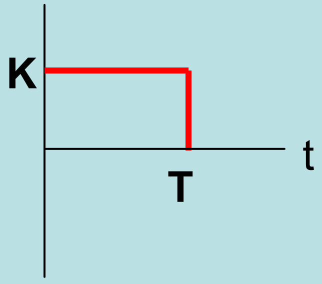
3.2. Representasi Geometris dari Sinyal – cont.

Contoh: M-ary PAM (M=4)



$m_1 = 00$, $m_2 = 01$ $m_3 = 10$, $m_4 = 11$

$\phi(t)$ *Fungsi Basis*



$$\int_0^T \phi_i^2(t)\,dt = 1$$

$$K = \sqrt{\frac{1}{T}}$$

3.2. Representasi Geometris dari Sinyal – cont.

Contoh: M-ary PAM (M=4) – cont.

Analisis (koefisien ekspansi):

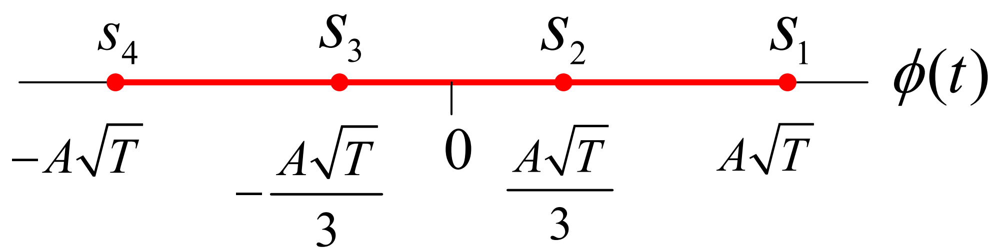
$$s_4 = \int_0^T s_1(t) \cdot \phi(t)\, dt = -A\sqrt{T}$$

$$s_3 = \int_0^T s_2(t) \cdot \phi(t)\, dt = -\frac{A\sqrt{T}}{3}$$

$$s_2 = \int_0^T s_3(t) \cdot \phi(t)\, dt = \frac{A\sqrt{T}}{3}$$

$$s_1 = \int_0^T s_4(t) \cdot \phi(t)\, dt = A\sqrt{T}$$

Representasi Geometris:



Signal Space (Konstelasi Sinyal) 1-Dimensi (1D)

- 1 Fungsi Basis
- $s_i(t)$ → s_i ~ sample

3.2. Representasi Geometris dari Sinyal – cont.

Point-point penting:

- Sinyal Waveform '*dipetakan*' menjadi Sinyal Vektor

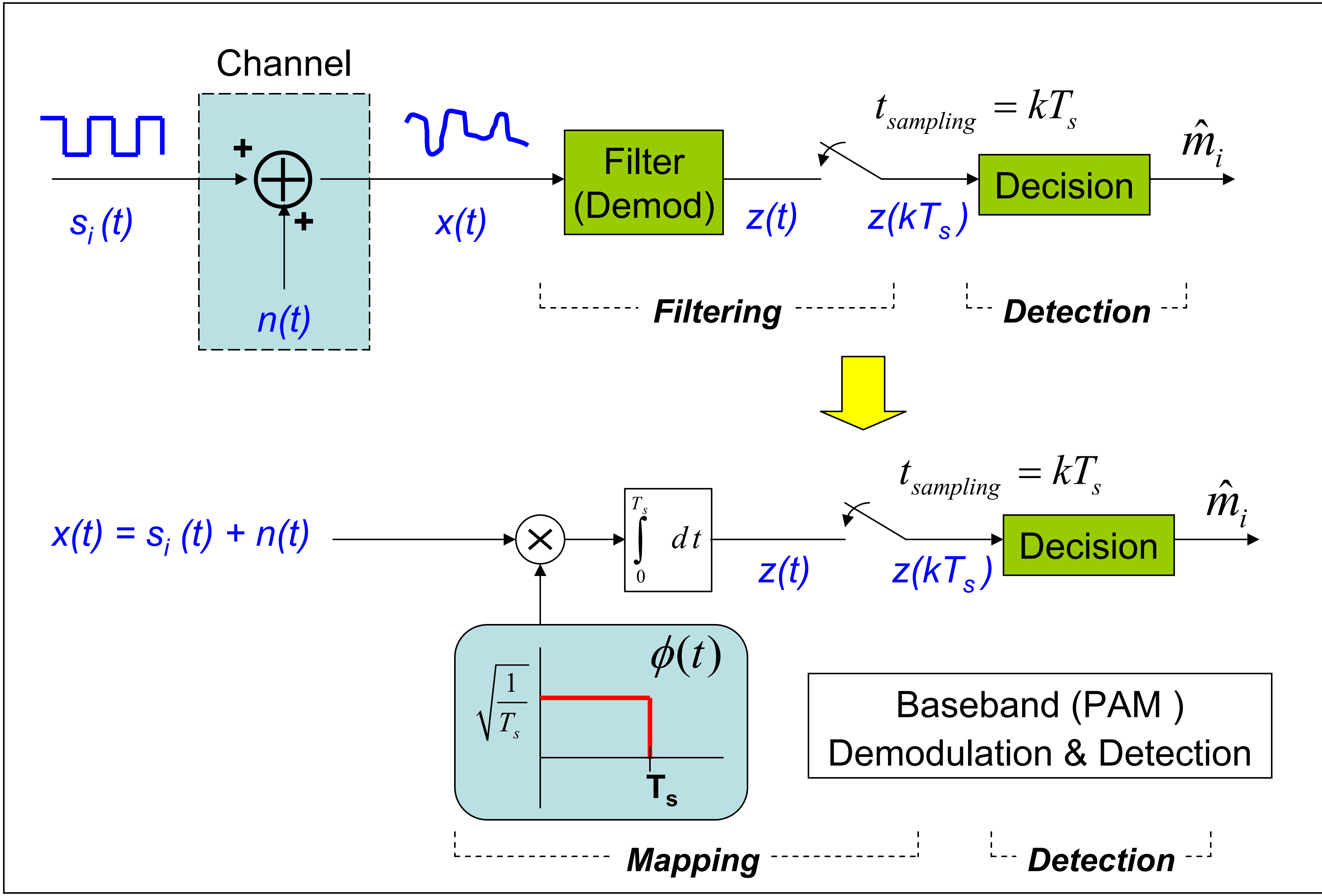
$$s_i(t) \longrightarrow \vec{s}_i \qquad ; i = 1, 2, \ldots, M$$

- Fungsi basis $\phi_j(t)$ $; j = 1, 2, \ldots N$ berperan sebagai fungsi pemetaaan tersebut.

- Fungsi basis bersifat Orthonormal:

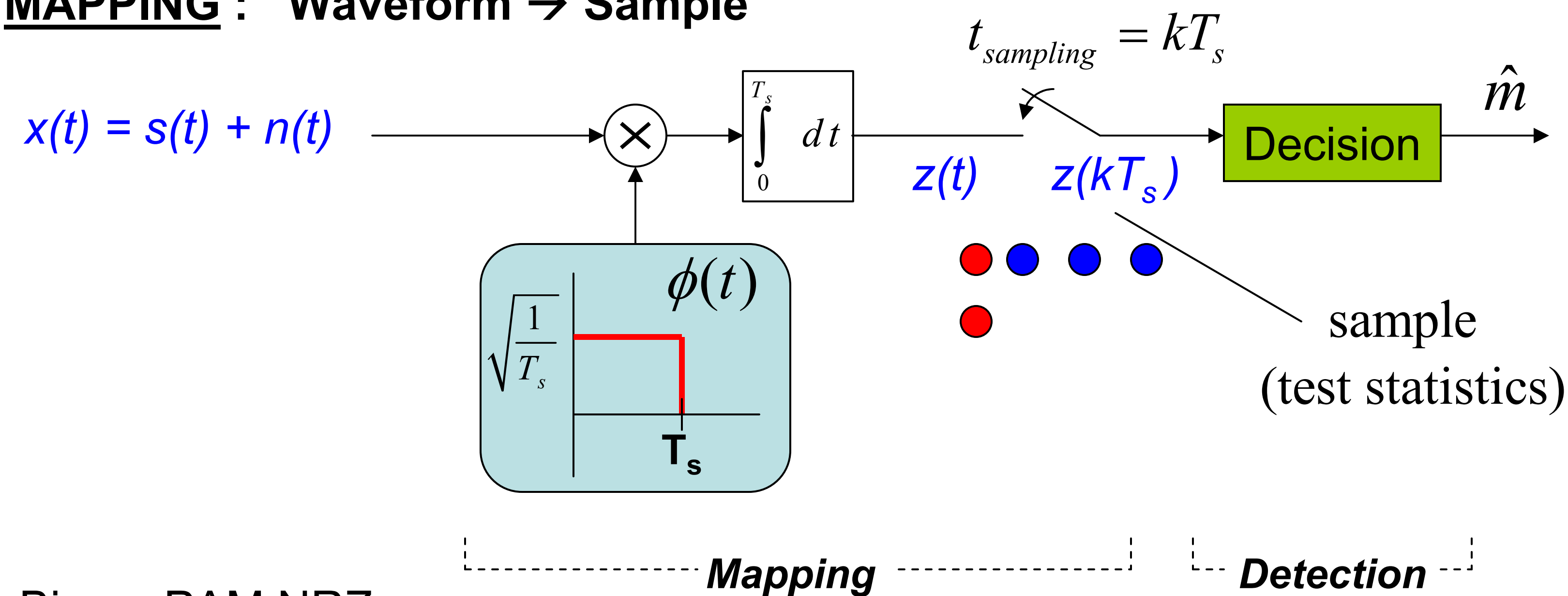
$$\int_0^T \phi_i(t) \cdot \phi_j(t)\, dt = \begin{cases} 1 & ; i = j \\ 0 & ; i \neq j \end{cases} \quad \Rightarrow \quad \int_0^T \phi_i^2(t)\, dt = 1$$

3.3. Optimal Detection: *"Maximum Likelihood Detection"*

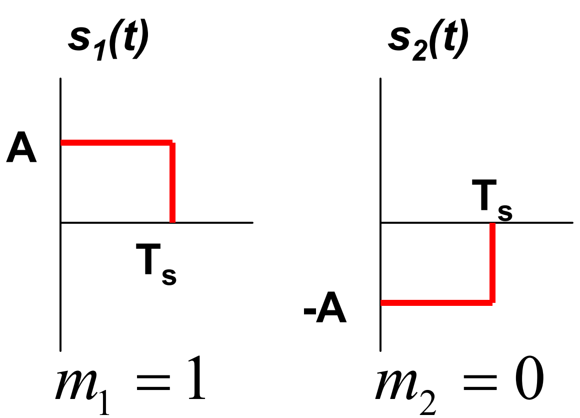


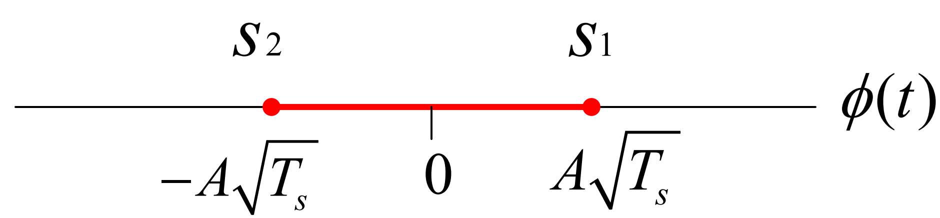
3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

MAPPING : Waveform → Sample



Binary PAM NRZ

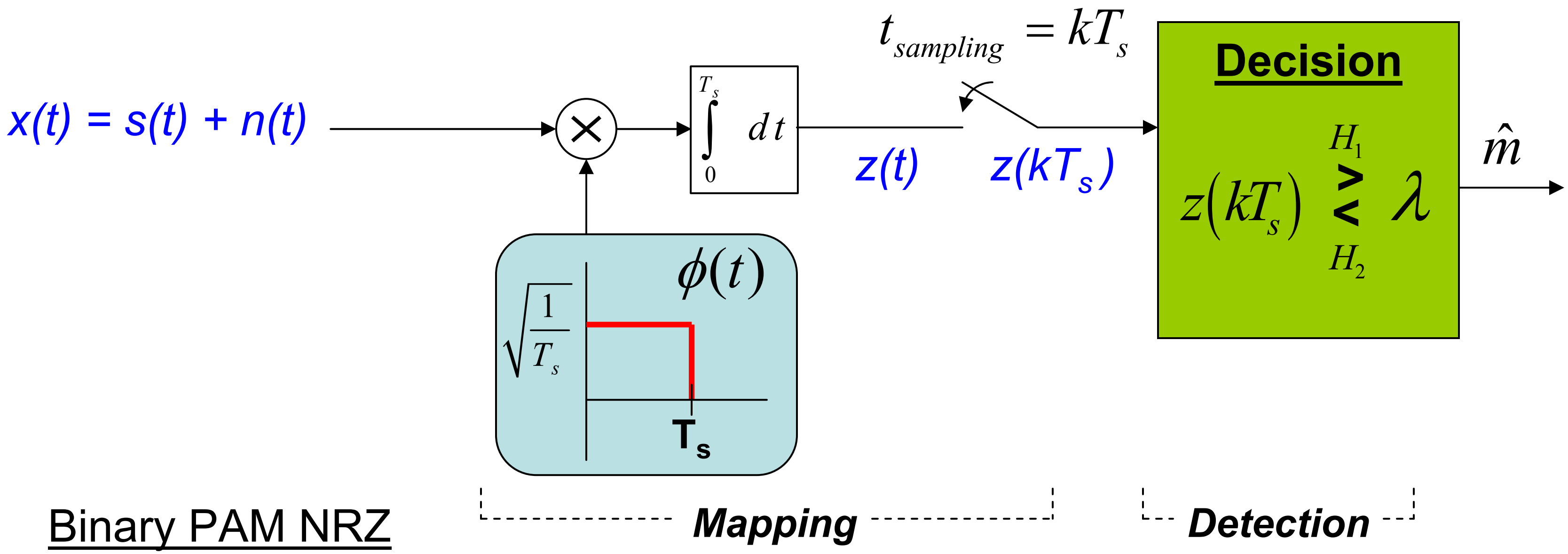




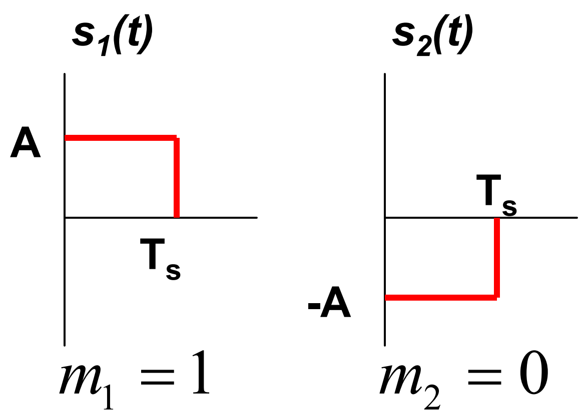
Konstelasi Sinyal (untuk Binary PAM NRZ):

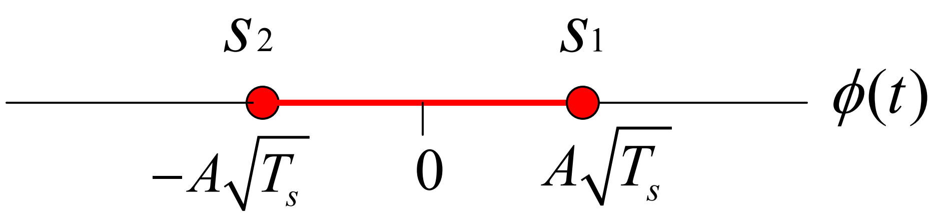
3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

DECISION : **Bandingkan test statistic VS. sebuah nilai threshold.**



Binary PAM NRZ





Konstelasi Sinyal (untuk Binary PAM NRZ):

3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

$$z(kT_s) = z(t)\Big|_{t=kT_s}$$

$$x(t) = s_i(t) + n(t)$$

$$= \int_0^{T_s} x(t)\,\phi(t)\,dt\Big|_{t=kT_s}$$

$$= \int_0^{T_s} s_i(t)\,\phi(t)\,dt\Big|_{t=kT_s} + \int_0^{T_s} n(t)\,\phi(t)\,dt\Big|_{t=kT_s}$$

$$= s_i(k) + n_0(k)$$

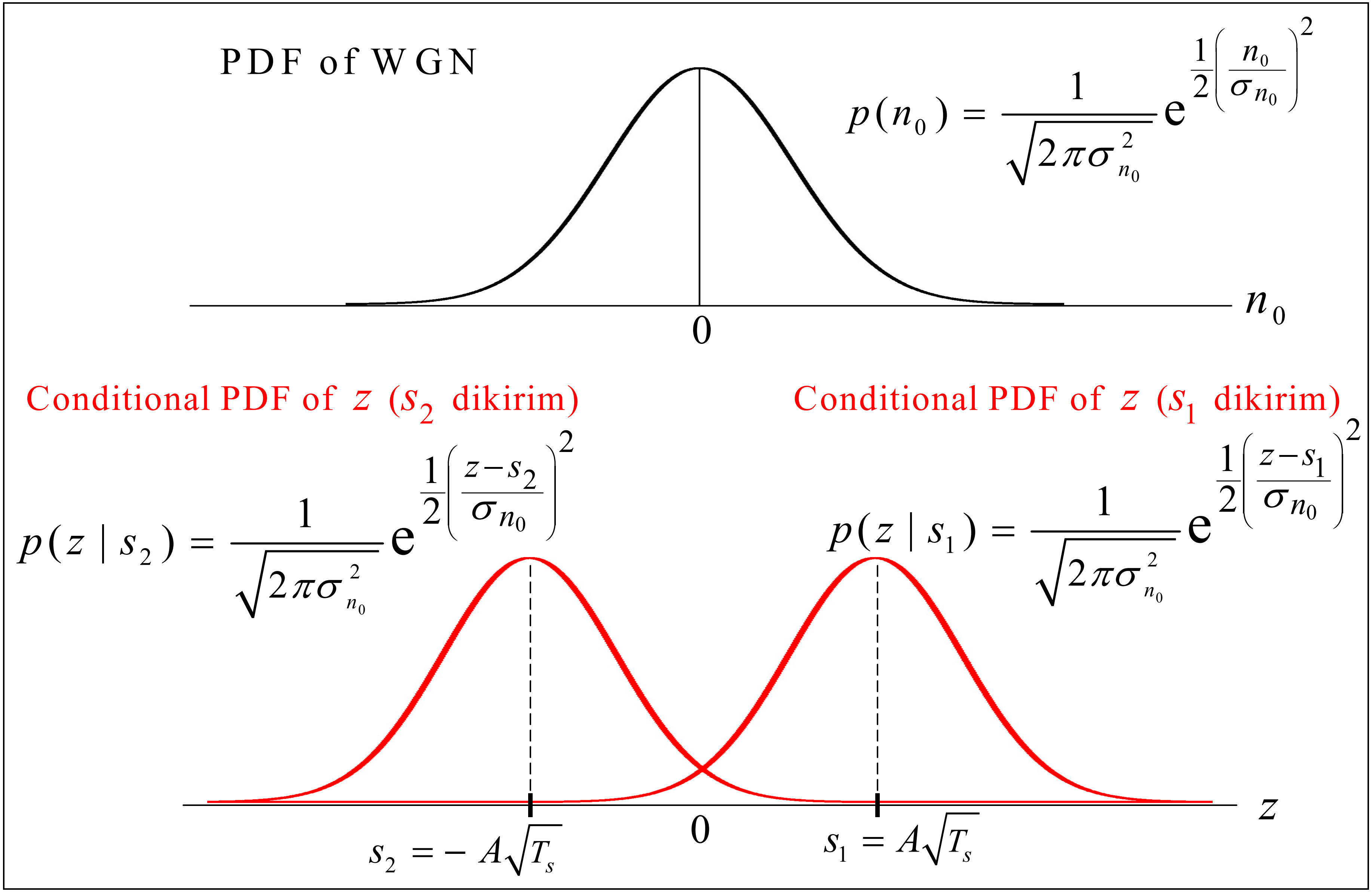
$$z = s_i + n_0$$

mean variance

gaussian random variable $\sim N(0, \sigma_n^2)$

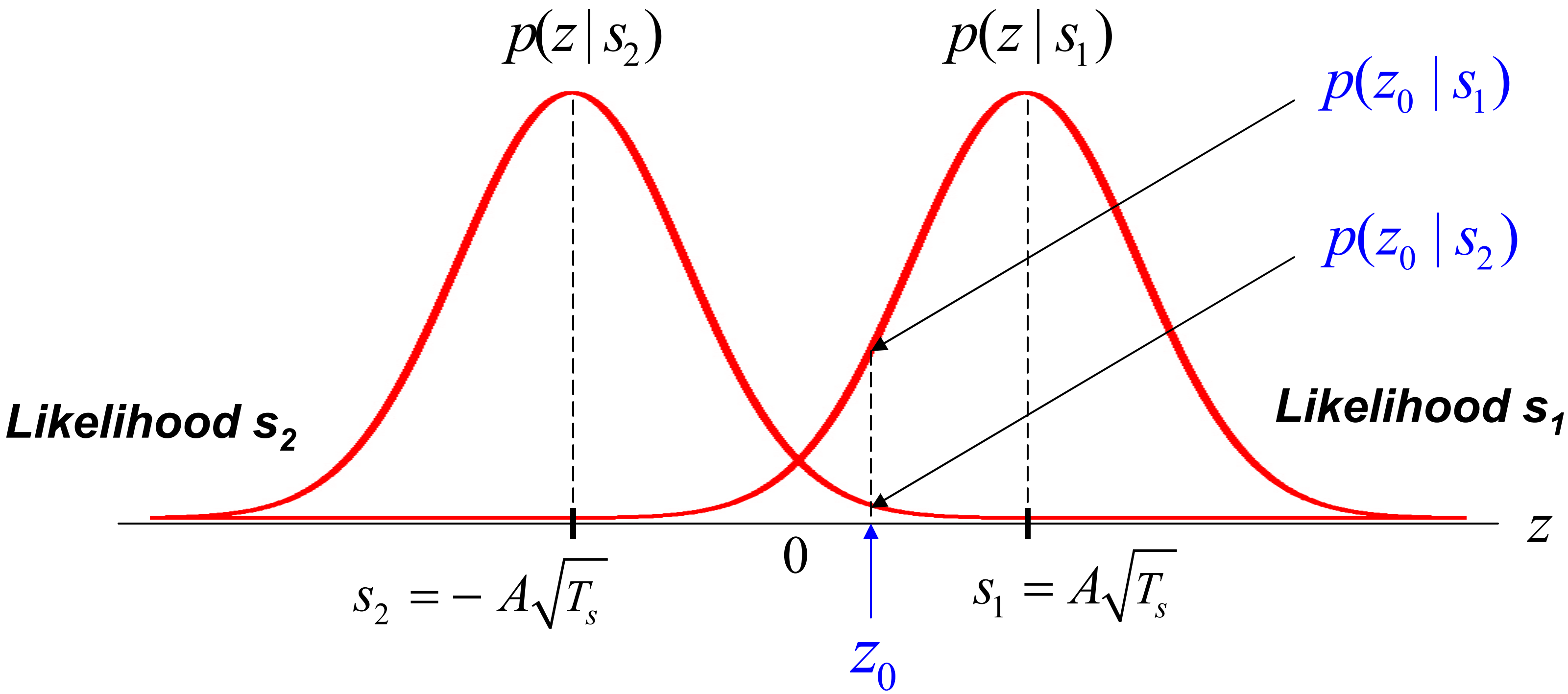
gaussian random variable $\sim N(s_i, \sigma_n^2)$

3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.



3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

Likelihood Ratio Test:



$$\Lambda(z_0) = \frac{p(z_0 \mid s_1)}{p(z_0 \mid s_2)} \frac{p(s_1)}{p(s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$H_1 = s_1$ dikirim.
$H_2 = s_2$ dikirim.

3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

Likelihood Ratio Test:

$$\Lambda(z_0) = \frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{p(s_1)}{p(s_2)} = 1$$

Untuk 'equi-probable' binary simbol digit:

$p(s_1) = p(s_2) = 1/2$

$$= \frac{\dfrac{1}{\sqrt{2\pi\sigma_{n_0}^2}} \exp\left\{\dfrac{1}{2}\left(\dfrac{z-s_1}{\sigma_{n_0}}\right)^2\right\}}{\dfrac{1}{\sqrt{2\pi\sigma_{n_0}^2}} \exp\left\{\dfrac{1}{2}\left(\dfrac{z-s_2}{\sigma_{n_0}}\right)^2\right\}} \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$$= \frac{\exp\left(-\dfrac{z_0^2}{2\sigma_{n_0}^2}\right)\cdot\exp\left(-\dfrac{s_1^2}{2\sigma_{n_0}^2}\right)\cdot\exp\left(-\dfrac{2z_0s_1}{2\sigma_{n_0}^2}\right)}{\exp\left(-\dfrac{z_0^2}{2\sigma_{n_0}^2}\right)\cdot\exp\left(-\dfrac{s_2^2}{2\sigma_{n_0}^2}\right)\cdot\exp\left(-\dfrac{2z_0s_2}{2\sigma_{n_0}^2}\right)} \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

Likelihood Ratio Test:

$$\Lambda(z_0) = \exp\left(\frac{z_0(s_1 - s_2)}{\sigma_{n_0}^2} - \frac{(s_1^2 - s_2^2)}{2\sigma_{n_0}^2}\right) \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$$\ln\left[\Lambda(z_0)\right] = \frac{z_0(s_1 - s_2)}{\sigma_{n_0}^2} - \frac{(s_1^2 - s_2^2)}{2\sigma_{n_0}^2} \underset{H_2}{\overset{H_1}{\gtrless}} \ln[1] = 0$$

$$\frac{z_0(s_1 - s_2)}{\sigma_{n_0}^2} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{(s_1^2 - s_2^2)}{2\sigma_{n_0}^2}$$

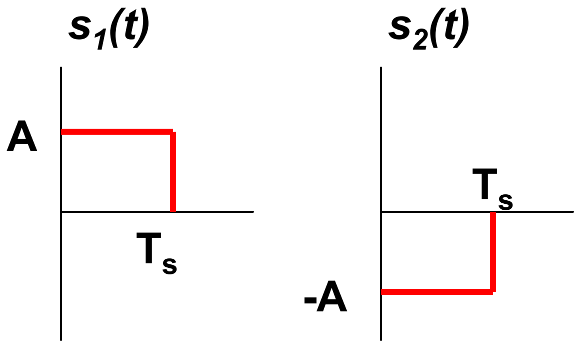
$$\boxed{z_0 \underset{H_2}{\overset{H_1}{\gtrless}} \frac{s_1 + s_2}{2}}$$

Maximum Likelihood (ML) Detection Rule untuk Transmisi Binary PAM

3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

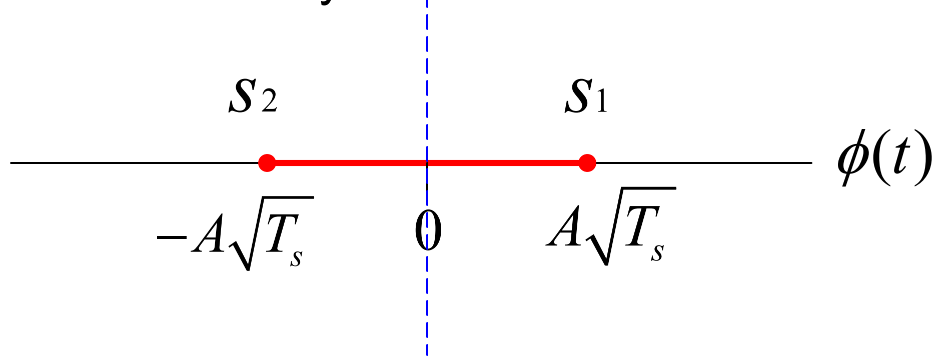
Likelihood Ratio Test:

Contoh: Binary PAM



$m_1 = 1$ $m_2 = 0$

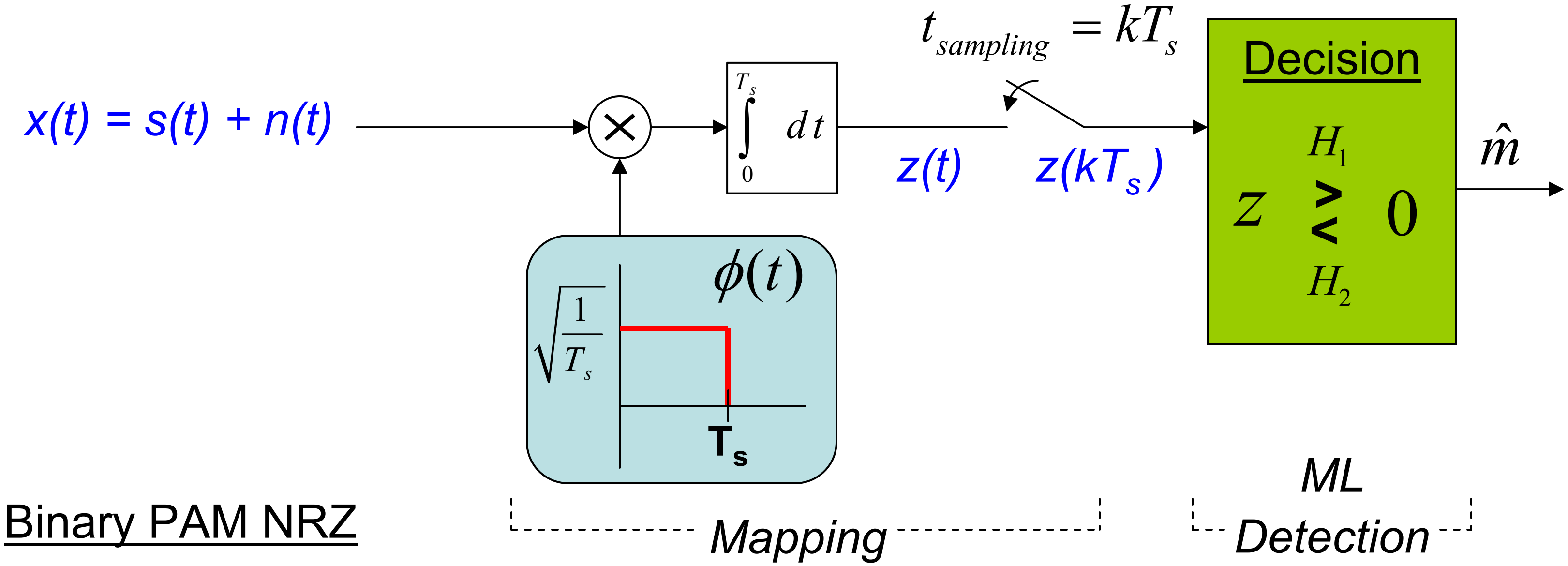
Konstelasi Sinyal:



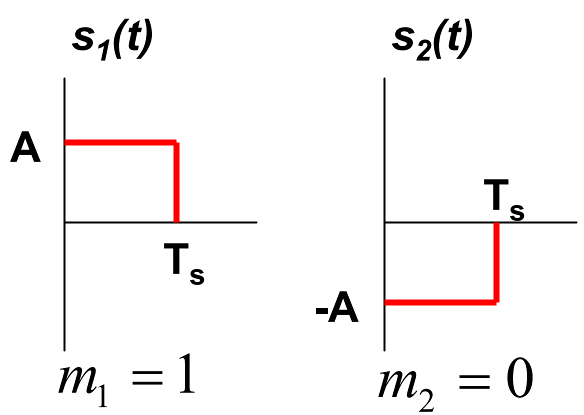
$$z_0 \underset{H_2}{\overset{H_1}{\gtrless}} \frac{s_1 + s_2}{2} = \frac{A\sqrt{T_s} + (-A\sqrt{T_s})}{2} = 0$$

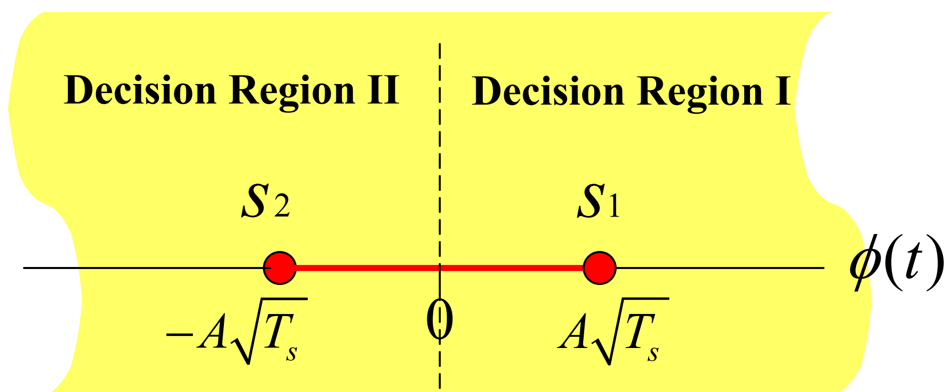
3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

(Optimal) Maximum Likelihood Detection untuk Binary PAM NRZ:



Binary PAM NRZ

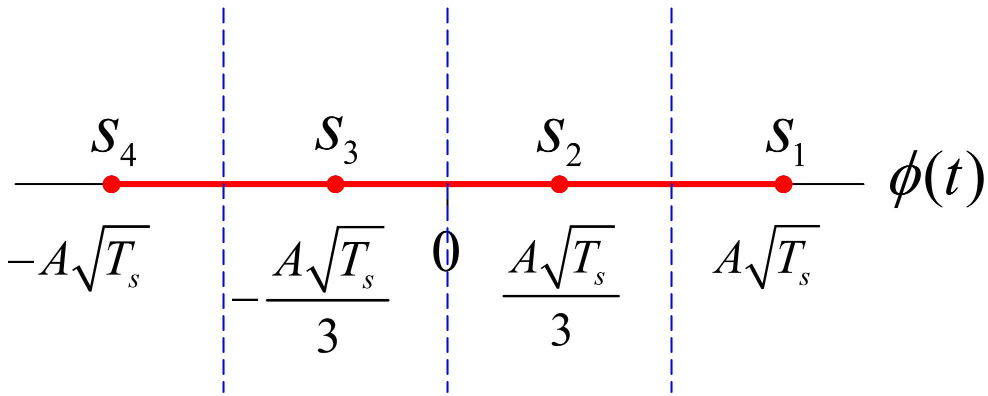




3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

Likelihood Ratio Test:

Contoh: **4-Ary PAM**



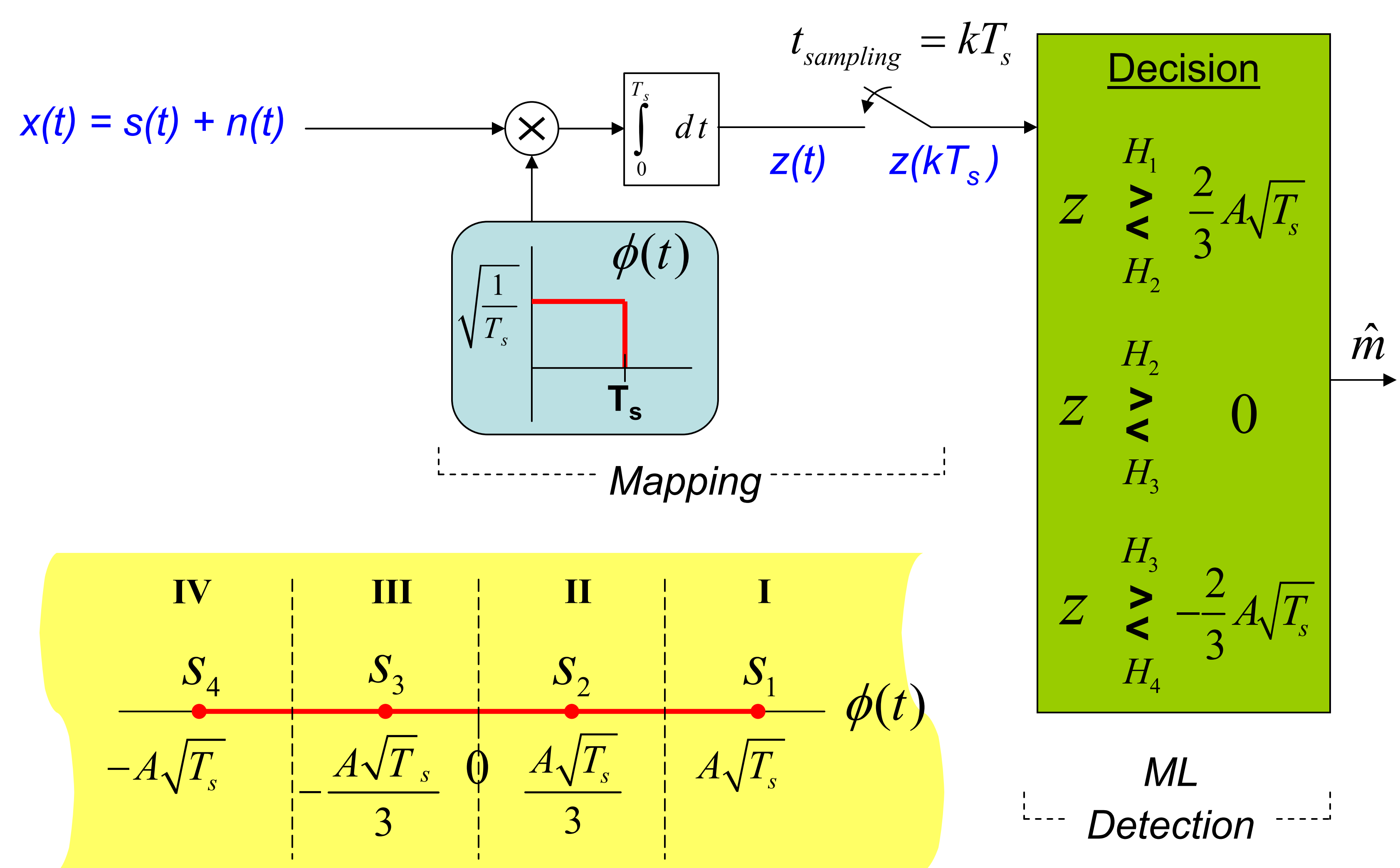
3 Nilai Threshold

$$z_0 \overset{H_1}{\underset{H_2}{\gtrless}} \frac{s_2 + s_1}{2} = \frac{A\sqrt{T_s}/3 + A\sqrt{T_s}}{2} = \frac{2}{3}A\sqrt{T_s}$$

$$z_0 \overset{H_2}{\underset{H_3}{\gtrless}} \frac{s_3 + s_2}{2} = \frac{-A\sqrt{T_s}/3 + A\sqrt{T_s}/3}{2} = 0$$

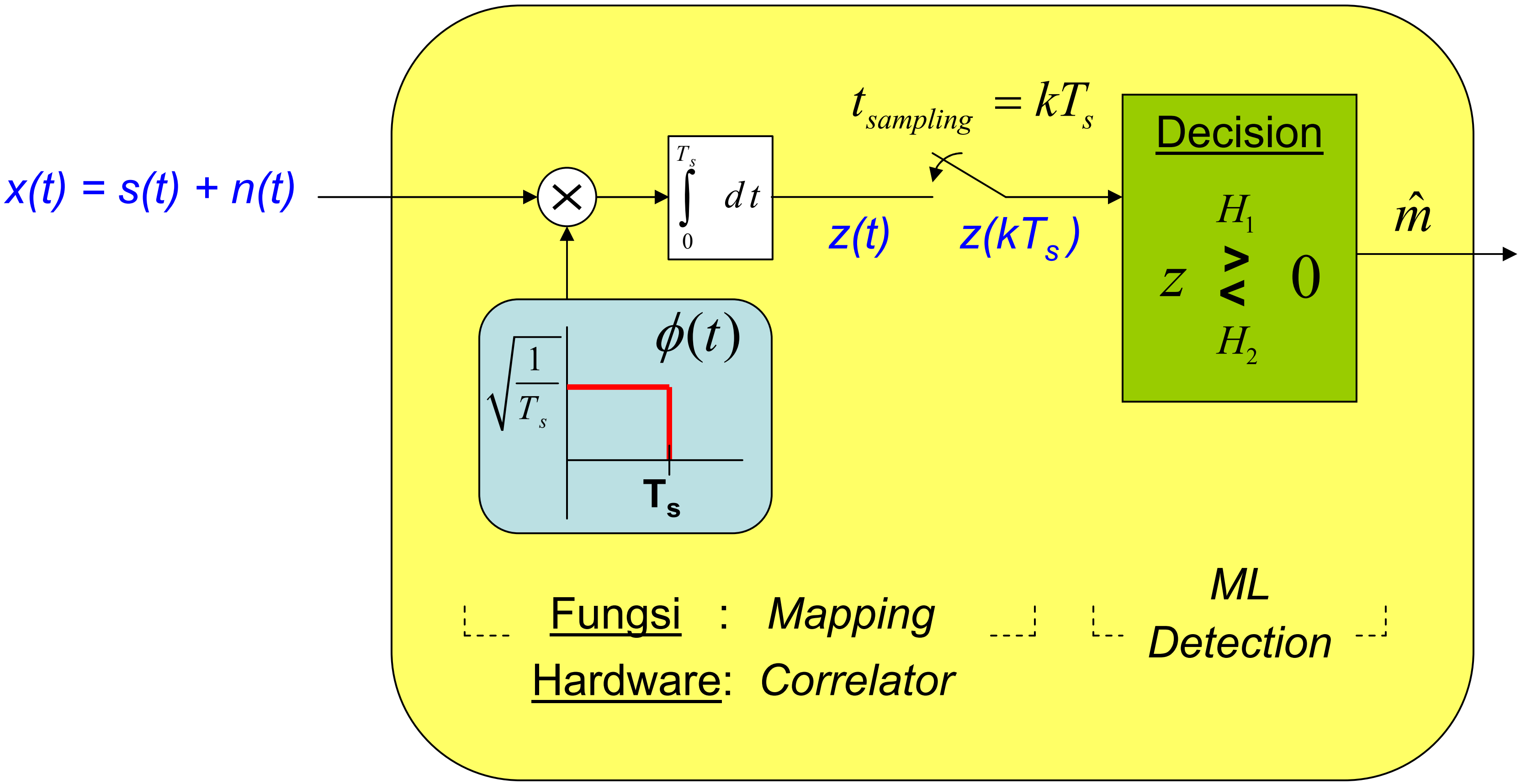
$$z_0 \overset{H_3}{\underset{H_4}{\gtrless}} \frac{s_4 + s_3}{2} = \frac{-A\sqrt{T_s} + (-A\sqrt{T_s}/3)}{2} = -\frac{2}{3}A\sqrt{T_s}$$

(Optimal) Maximum Likelihood Detection untuk M-ary PAM:



3.3. Optimal Detection: *"Maximum Likelihood Detection"* – cont.

Correlator Receiver dengan ML Detection untuk Binary PAM NRZ



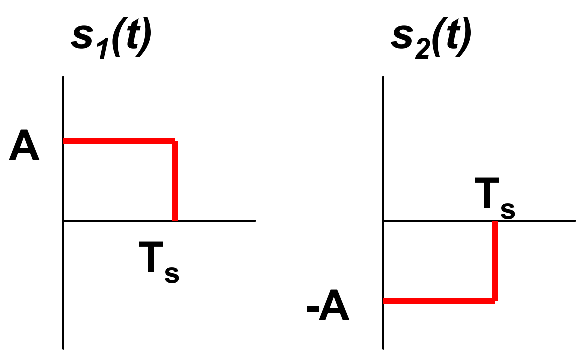
3.4. Energy/Symbol, Energy/Bit, dan Minimum Distance.

Energy/Symbol, $E_s = \frac{1}{M}\sum_{k=1}^{M} E_{s_k}$;M = Jumlah simbol di dalam alfabet.

Energy/Bit, $E_b = \frac{T_b}{T_s} E_s$

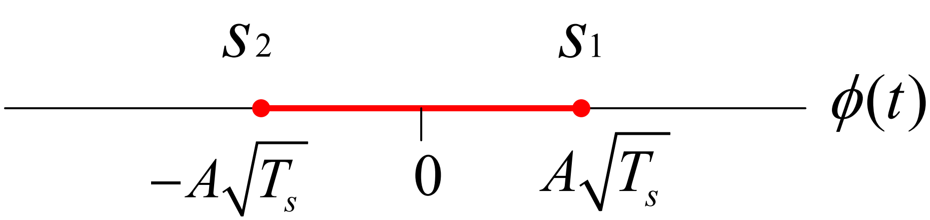
Minimum Distance, $D_{\min} =$ jarak antara 2 simbol yang terdekat.

Contoh: Binary PAM NRZ



$m_1 = 1$ $m_2 = 0$

Konstelasi Sinyal:

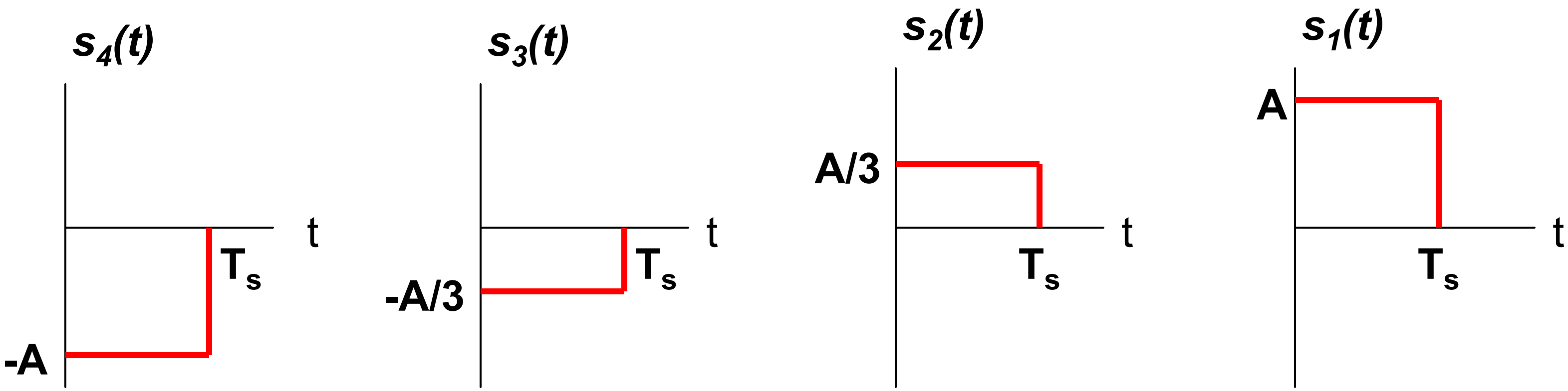


$E_b = E_s = A^2 T_s$, $D_{\min} = 2A\sqrt{T_s}$

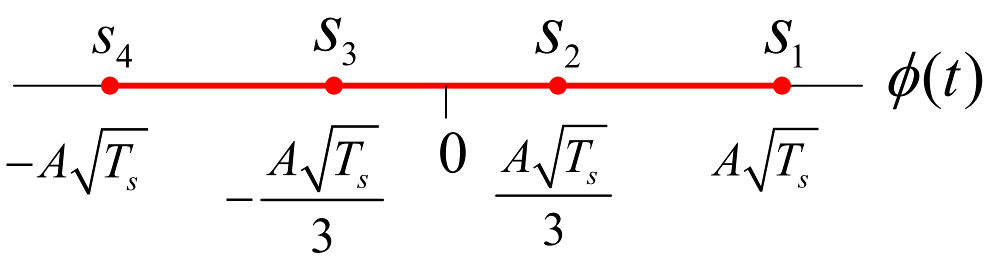
$= 2\sqrt{E_b}$

3.4. Energy/Symbol, Energy/Bit, dan Minimum Distance – cont.

Contoh: 4-ary PAM



Konstelasi Sinyal:



$$E_s = \frac{5}{9}A^2T_s \quad , \quad E_b = \frac{1}{2}E_s = \frac{5}{18}A^2T_s \quad , \quad D_{\min} = \frac{2}{3}A\sqrt{T_s}$$

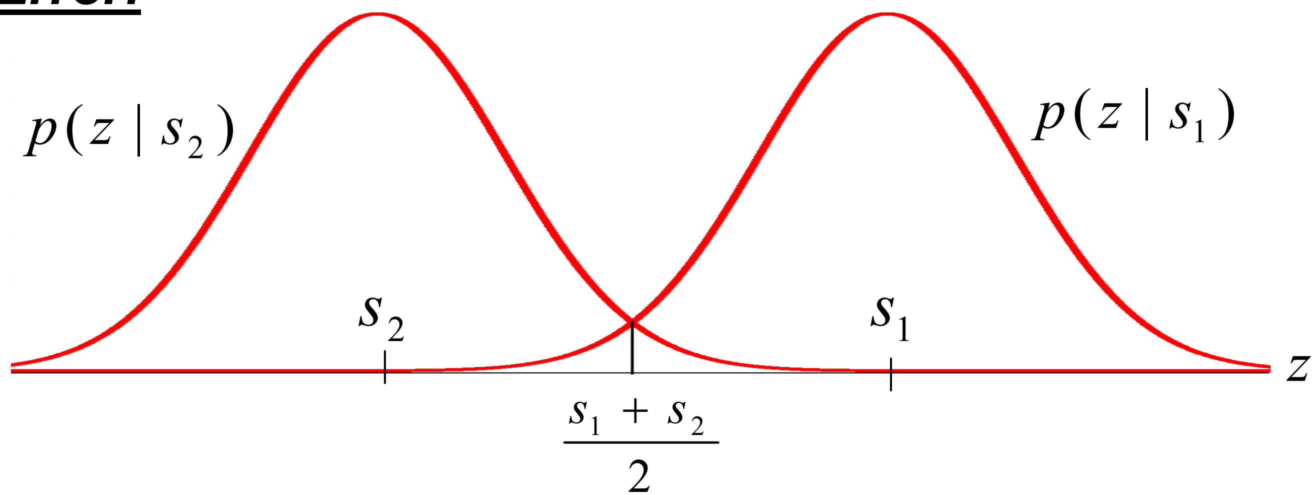
$$= 2\sqrt{\frac{2}{5}E_b}$$

3.5. Probabilitas Error untuk Transmisi Binary PAM dengan (Optimal) Maximum Likelihood Detection

Probabilitas Error:

ML Detection

$$z \underset{H_2}{\overset{H_1}{\gtrless}} \frac{s_1 + s_2}{2}$$



$$P(e \mid s_1) = \int_{-\infty}^{z=\frac{s_1+s_2}{2}} p(z \mid s_1)\, dz \qquad P(e \mid s_2) = \int_{z=\frac{s_1+s_2}{2}}^{\infty} p(z \mid s_2)\, dz$$

$$P_e = P(e \mid s_1) \cdot P(s_1) + P(e \mid s_2) \cdot P(s_2)$$ - Probabilitas Total Rata2

$$= \frac{1}{2}\left[P(e \mid s_1) + P(e \mid s_2)\right]$$ - equi-probable simbol digit

$$= P(e \mid s_2) = P(e \mid s_1)$$ - conditional PDF simetrik

3.5. Probabilitas Error untuk Transmisi Binary PAM dengan (Optimal) Maximum Likelihood Detection – cont.

Probabilitas Error:

$$P_e = \int_{z=\frac{s_1+s_2}{2}}^{\infty} p(z\,|\,s_2)\,dz = \int_{z=\frac{s_1+s_2}{2}}^{\infty} \frac{1}{\sigma_{n_0}\sqrt{2\pi}} \exp\left[\frac{1}{2}\left(\frac{z-s_2}{\sigma_{n_0}}\right)^2\right] dz$$

$$u = \left(\frac{z-s_2}{\sigma_{n_0}}\right) \quad \Rightarrow \quad \frac{du}{dz} = \frac{1}{\sigma_{n_0}} \quad \rightarrow \quad \sigma_{n_0}\,du = dz$$

- pergantian variabel

$$= \int_{u=\frac{s_1-s_2}{2\sigma_{n_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{1}{2}u^2\right] du$$

$$\boxed{P_e = \int_{\frac{s_1-s_2}{2\sigma_{n_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{1}{2}u^2\right] du = Q\left(\frac{s_1-s_2}{2\sigma_{n_0}}\right)}$$

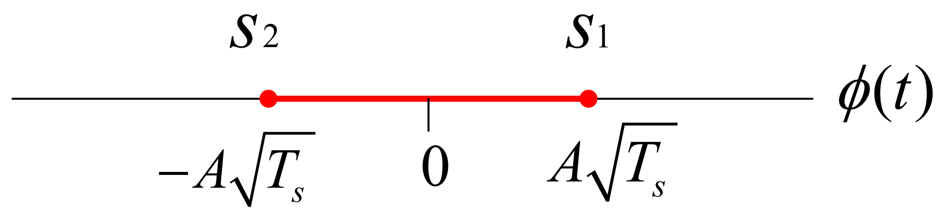
- Complementary Error Function (Q-Function)

- <u>ditabulasikan</u> -

3.5. Probabilitas Error untuk Transmisi Binary PAM dengan (Optimal) Maximum Likelihood Detection – cont.

Probabilitas Error:

Contoh: Binary PAM



$$P_e = Q\left(\frac{s_1 - s_2}{2\sigma_{n_0}}\right)$$

$$= Q\left(\frac{A\sqrt{T_s} - (-A\sqrt{T_s})}{2\sigma_{n_0}}\right)$$

$$= Q\left(\frac{A\sqrt{T_s}}{\sigma_{n_0}}\right) = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\frac{D_{min}}{\sqrt{2N_0}}\right)$$

Bit-Error Rate (BER)

Symbol-Error Rate (SER)

3.6. Optimal Filter: "Matched Filter" or "Correlator"

***Kriteria optimal untuk filtering*:**

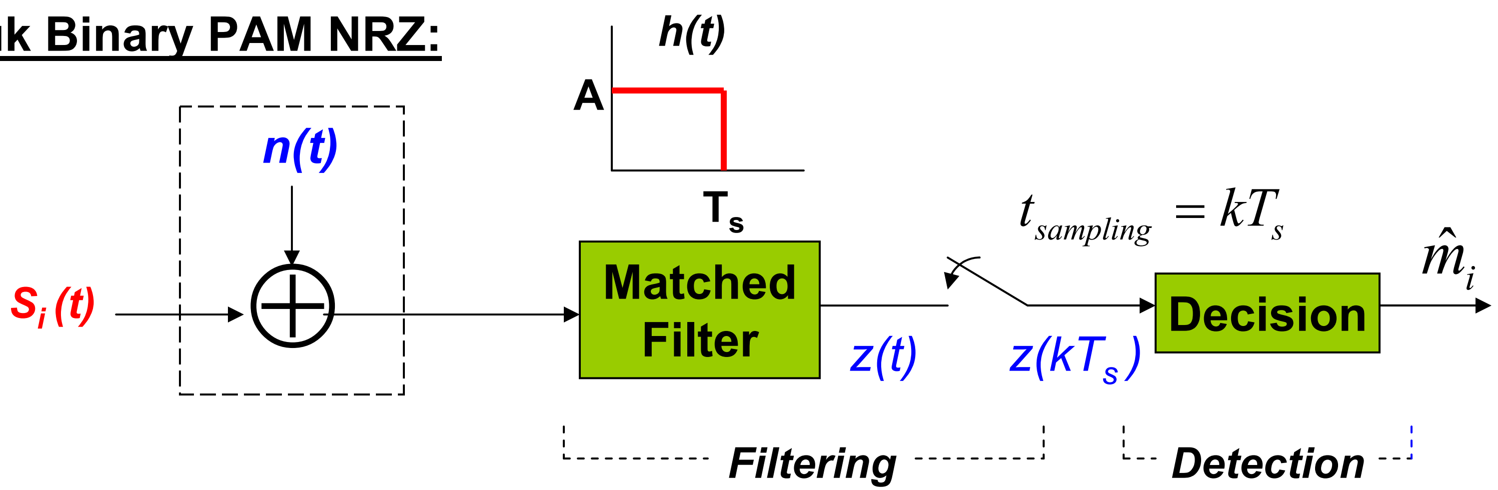
Bentuk demodulator filter yang optimal adalah filter yang me-maksimalkan Signal-to-Noise Power Ratio (SNR) pada output-nya.

Filter yang memenuhi kriteria di atas: ***Matched Filter***

Respon Impuls: $h(t) = s(T_s - t)$

,dimana $s(t)$ adalah sinyal input, T_s adalah durasi dari s(t).

Untuk Binary PAM NRZ:

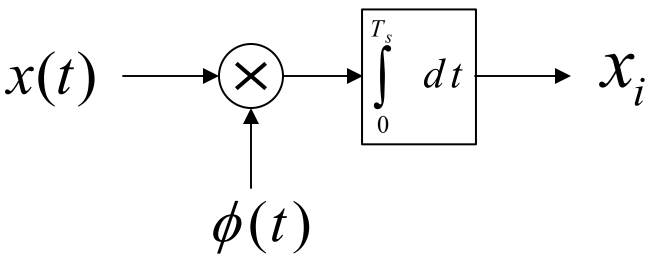


3.6. Optimal Filter: "Matched Filter" or "Correlator" – cont.

***Matched Filter* sebagai *Correlator* – cont.**

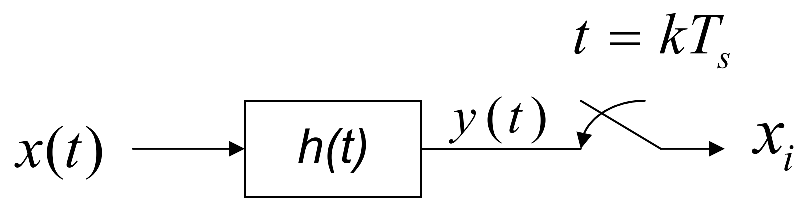
Ekuivalensi antara *Correlator* dan *Matched Filter* :

Correlator



$$x_i = \int_0^{T_s} x(t) \cdot \phi(t)\, dt$$

Matched Filter



$$h(t) = \phi(T_s - t)$$

$$y(\tau) = \int x(t) \cdot h(\tau - t)\, dt$$

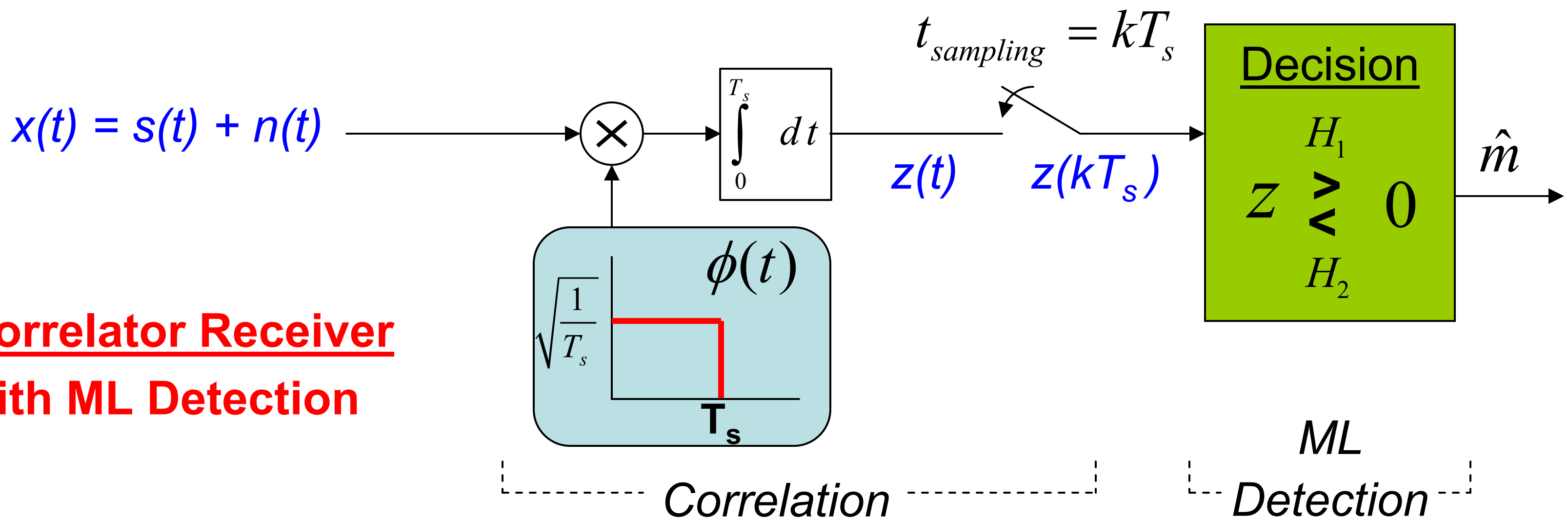
$$= \int x(t) \cdot \phi(\tau - T_s + t)\, dt$$

$$y(T_s) = \int x(t) \cdot \phi(t)\, dt = x_i$$

3.6. Optimal Filter: “Matched Filter” or “Correlator” – cont.

Optimal Receiver dengan ML Detection untuk Binary PAM NRZ:

Correlator Receiver with ML Detection



Matched Filter Receiver with ML Detection