Sistem Komunikasi II
(*Digital Communication Systems*)

Lecture #6: Modulasi & Demodulasi Bandpass
(*Bandpass Modulation & Demodulation*)
- PART II –

Topik:

6.1 M-Frequency Shift Keying (M-FSK).
- *Modulasi, Transmitter, & Receiver (Coherent & Non-Coherent).*
- *Minimum Tone spacing & Bandwidth Transmisi.*

6.2 Probabilitas Simbol Error untuk M-FSK.

6.3 M-Quadrature Amplitude Modulation (M-QAM)
- *Modulasi, Transmitter, Receiver (Coherent).*

6.4 Probabilitas Error untuk M-QAM.

6.5 Perbandingan antara M-PSK, M-FSK, & M-QAM.

6.1. M-Frequency Shift Keying (M-FSK)

Modulasi M-FSK:

$$m_i \longrightarrow s_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos(\omega_i t) \quad ; 0 \le t \le T_s$$

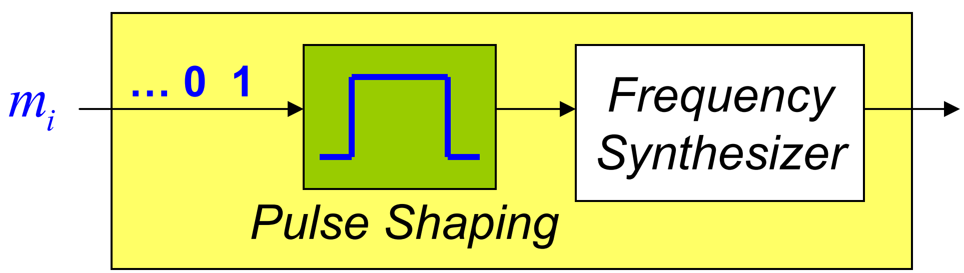
$$; i = 0, 2, \ldots, M-1$$

BFSK Transmitter:

M = 2 (Binary FSK - BFSK)

$$m_1 = 1 \longrightarrow s_1(t) = A\cos(\omega_1 t)$$

$$m_2 = 0 \longrightarrow s_2(t) = A\cos(\omega_2 t)$$

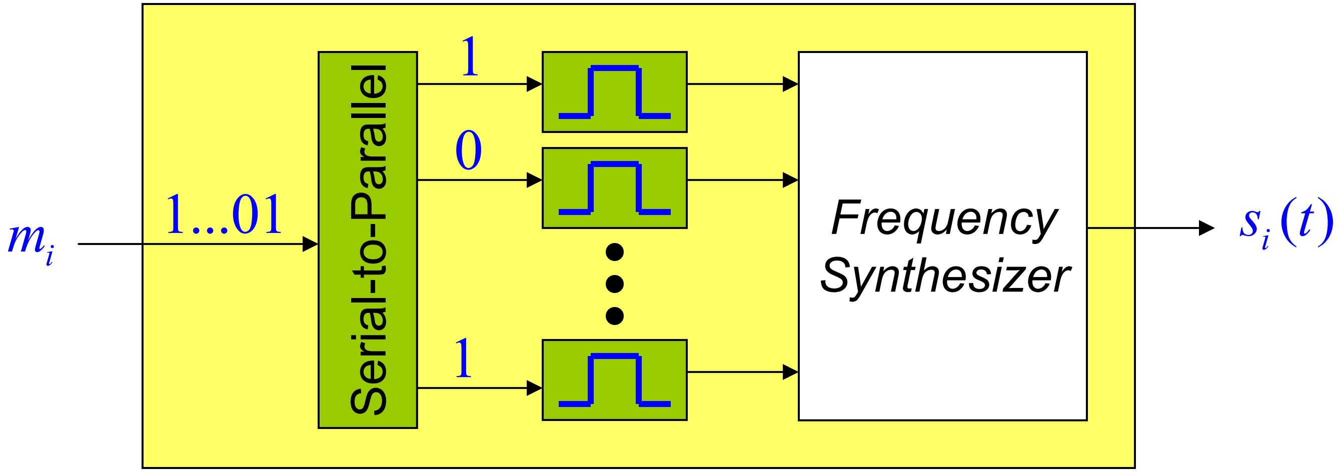


$$s_i(t) = \begin{cases} A\cos(\omega_1 t) & ; m_i = 1 \\ A\cos(\omega_2 t) & ; m_i = 0 \end{cases}$$

BFSK Modulator

6.1. M-Frequency Shift Keying (M-FSK) – cont.

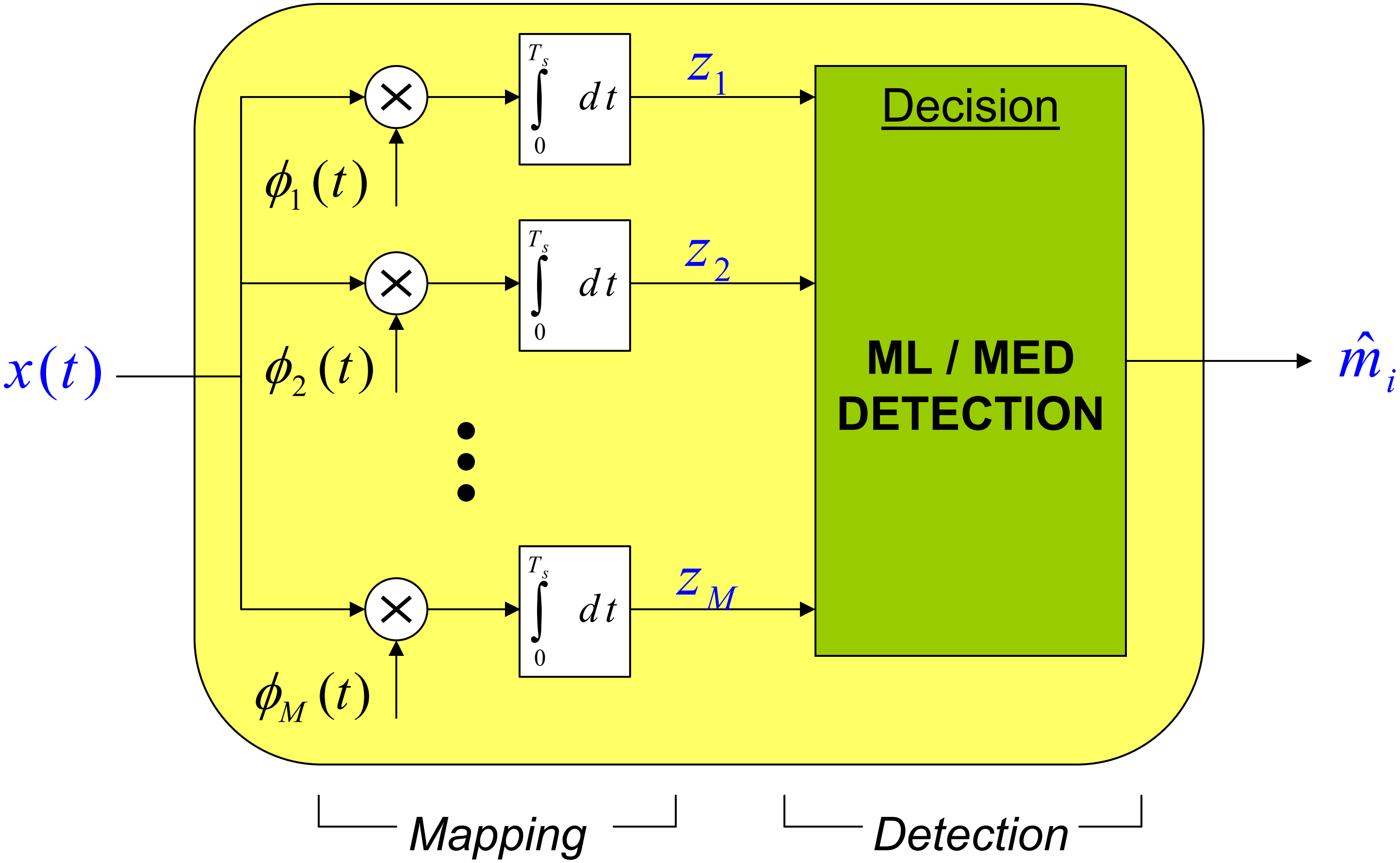
General M-FSK Transmitter:



Frequency Synthesizer **adalah suatu perangkat elektronik yang mampu membangkitkan sinyal dengan frekwensi dan fasa yang berbeda-beda. Frekwensi dan fasa sinyal outputnya dikontrol secara digital oleh nilai binary inputnya.**

6.1. M-Frequency Shift Keying (M-FSK) – cont.

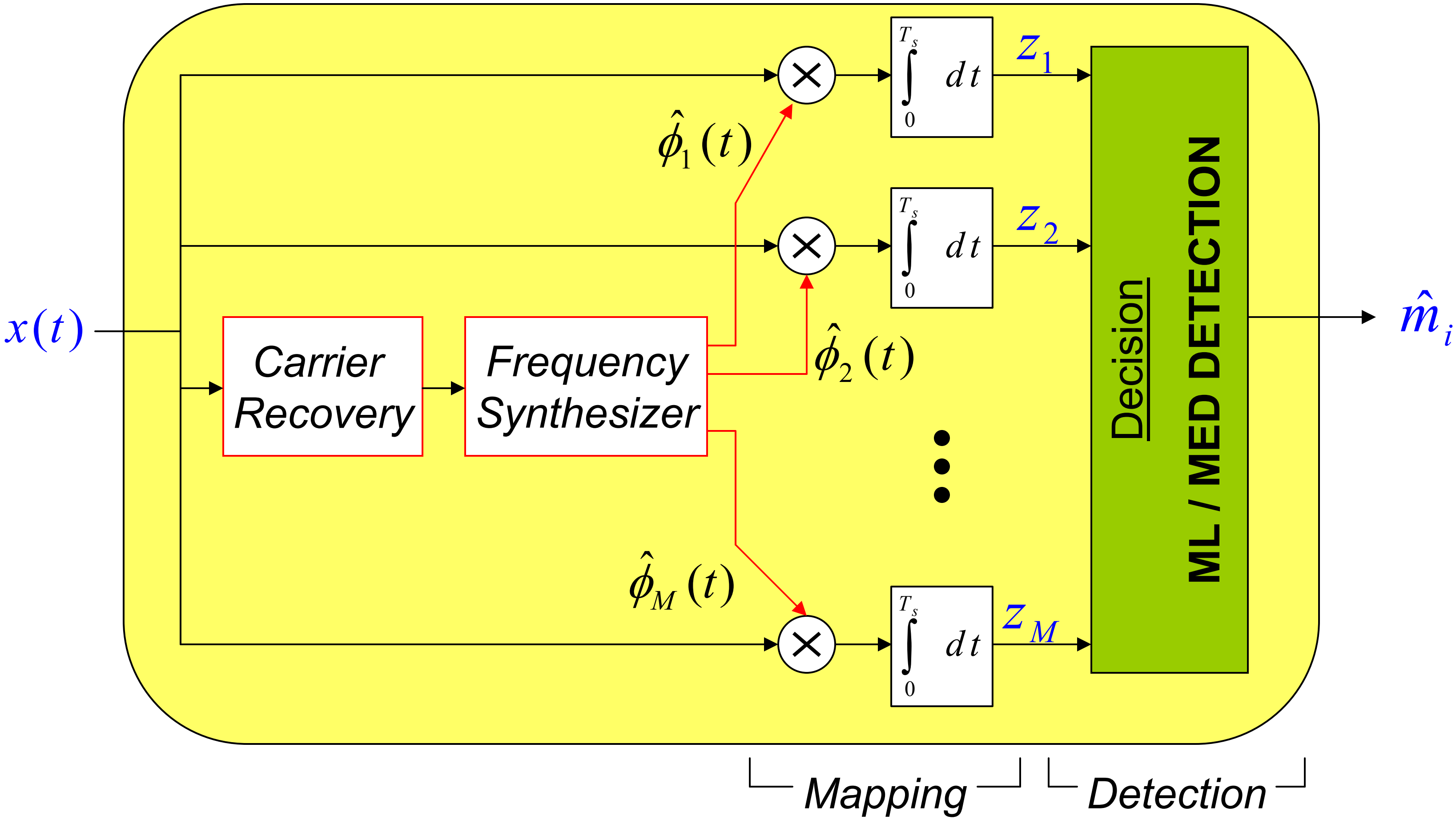
M-FSK Optimal (Coherent) Receiver:



$\phi_k(t) = \sqrt{2/T_s}\cos(\omega_k t + \alpha);$ α = pergeseran fasa akibat delay propagasi.

6.1. M-Frequency Shift Keying (M-FSK) – cont.

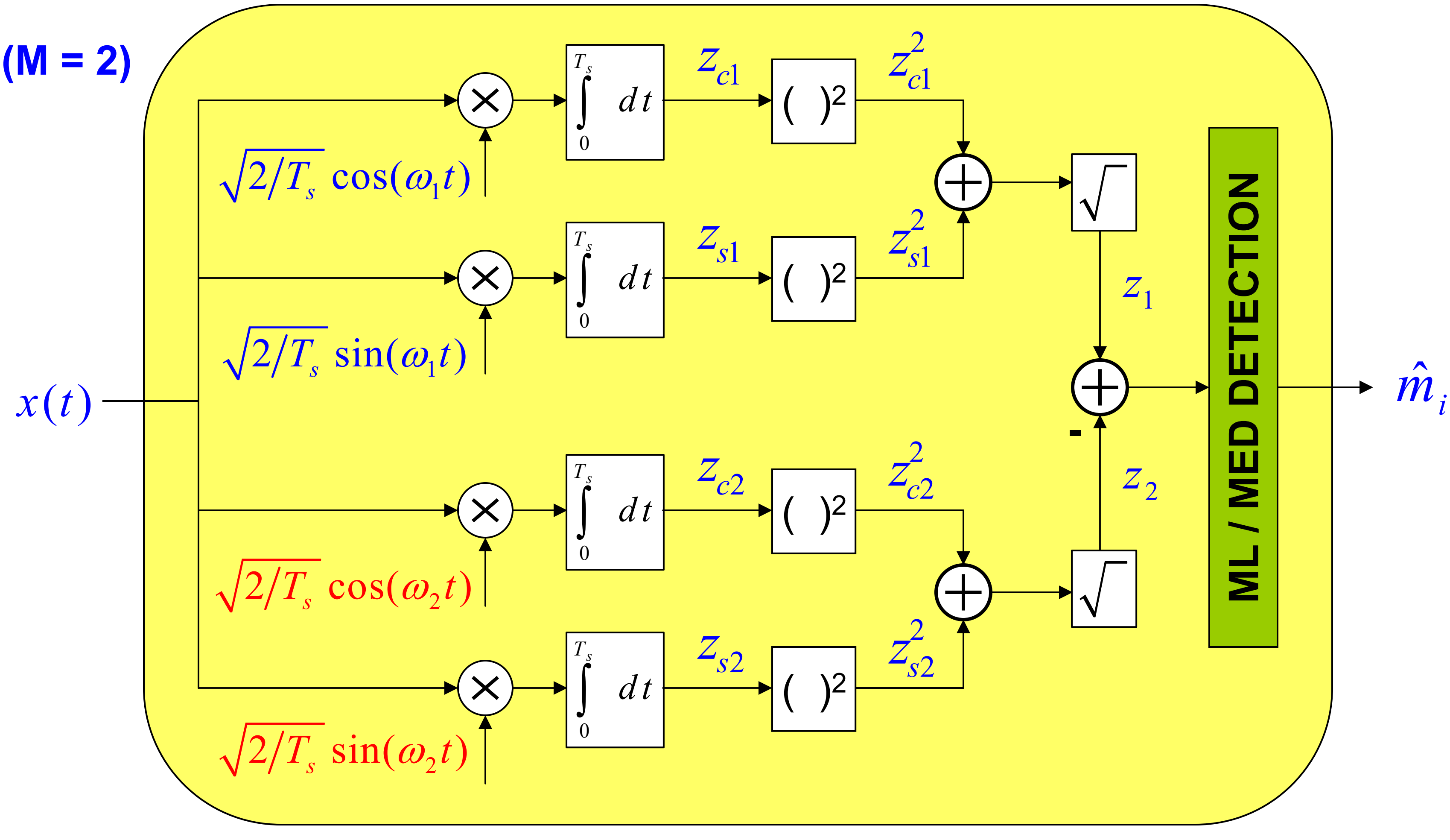
M-FSK Optimal (Coherent) Receiver – Implementasi Aktual:



$$\hat{\phi}_k(t) = \sqrt{2/T_s}\cos(\omega_k t + \hat{\alpha}) \qquad ;\alpha = \text{estimasi dari pergeseran fasa.}$$

6.1. M-Frequency Shift Keying (M-FSK) – cont.

M-FSK Non-Coherent (sub-optimal) Receiver: Energy Detector



* Non-Coherent receiver tdk membutuhkan informasi nilai α.

6.1. M-Frequency Shift Keying (M-FSK) – cont.

Minimum Tone Spacing. $(f_2 - f_1)_{\min}$

Minimum Tone spacing = jarak terkecil antara 2 frekwensi carrier yang diperbolehklan sehingga sinyal M-FSK yang dihasilkan bersifat Orthogonal.

Untuk Non-Coherent Receiver:

$$\int_0^{T_s} \cos(2\pi f_1 t + \theta)\cos(2\pi f_2 t)\, dt = 0$$ ← kriteria

⋮ ↓

$$\underbrace{\cos(\theta)\sin 2\pi(f_2 - f_1)T_s}_{=0} + \underbrace{\sin(\theta)[\cos 2\pi(f_2 - f_1)T_s - 1]}_{=0} = 0$$

$$\left.\begin{aligned} \sin 2\pi(f_2 - f_1)T_s &= 0 \\ \cos 2\pi(f_2 - f_1)T_s &= 1 \end{aligned}\right] \rightarrow 2\pi(f_2 - f_1)T_s = 2\pi k \Rightarrow (f_2 - f_1) = \frac{k}{T_s}$$

$$\boxed{(f_2 - f_1)_{\min} = \frac{1}{T_s} \text{ (Hz)}}$$

6.1. M-Frequency Shift Keying (M-FSK) – cont.

Minimum Tone Spacing. $(f_2 - f_1)_{\min}$

Untuk Coherent Receiver:

$$\int_0^{T_s} \cos\left(2\pi f_1 t + \theta\right)\cos\left(2\pi f_2 t + \theta\right) dt = 0$$ ← kriteria

⋮

$$\sin 2\pi (f_2 - f_1) T_s = 0$$

↓

$$2\pi (f_2 - f_1) T_s = \pi k$$

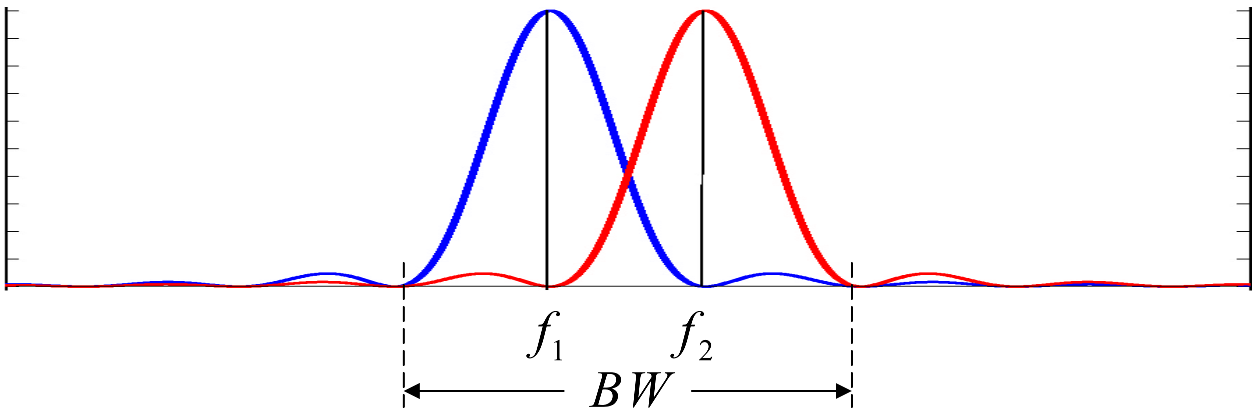
⇓

$$(f_2 - f_1) = \frac{k}{2T_s}$$ ⇒ $$(f_2 - f_1)_{\min} = \frac{1}{2T_s} \text{ (Hz)}$$

6.1. M-Frequency Shift Keying (M-FSK) – cont.

Bandwidth Transmisi B-FSK:

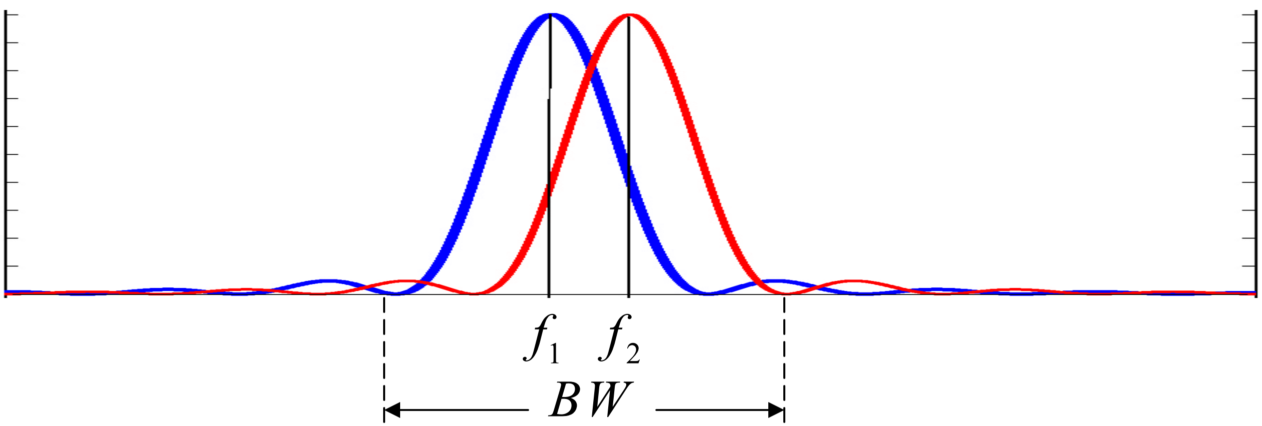
B-FSK dgn Non-Coherent Receiver



$$(f_2 - f_1)_{min} = \frac{1}{T_s}$$

$$BW = \frac{3}{T_s}$$

B-FSK dgn Coherent Receiver



$$(f_2 - f_1)_{min} = \frac{1}{2T_s}$$

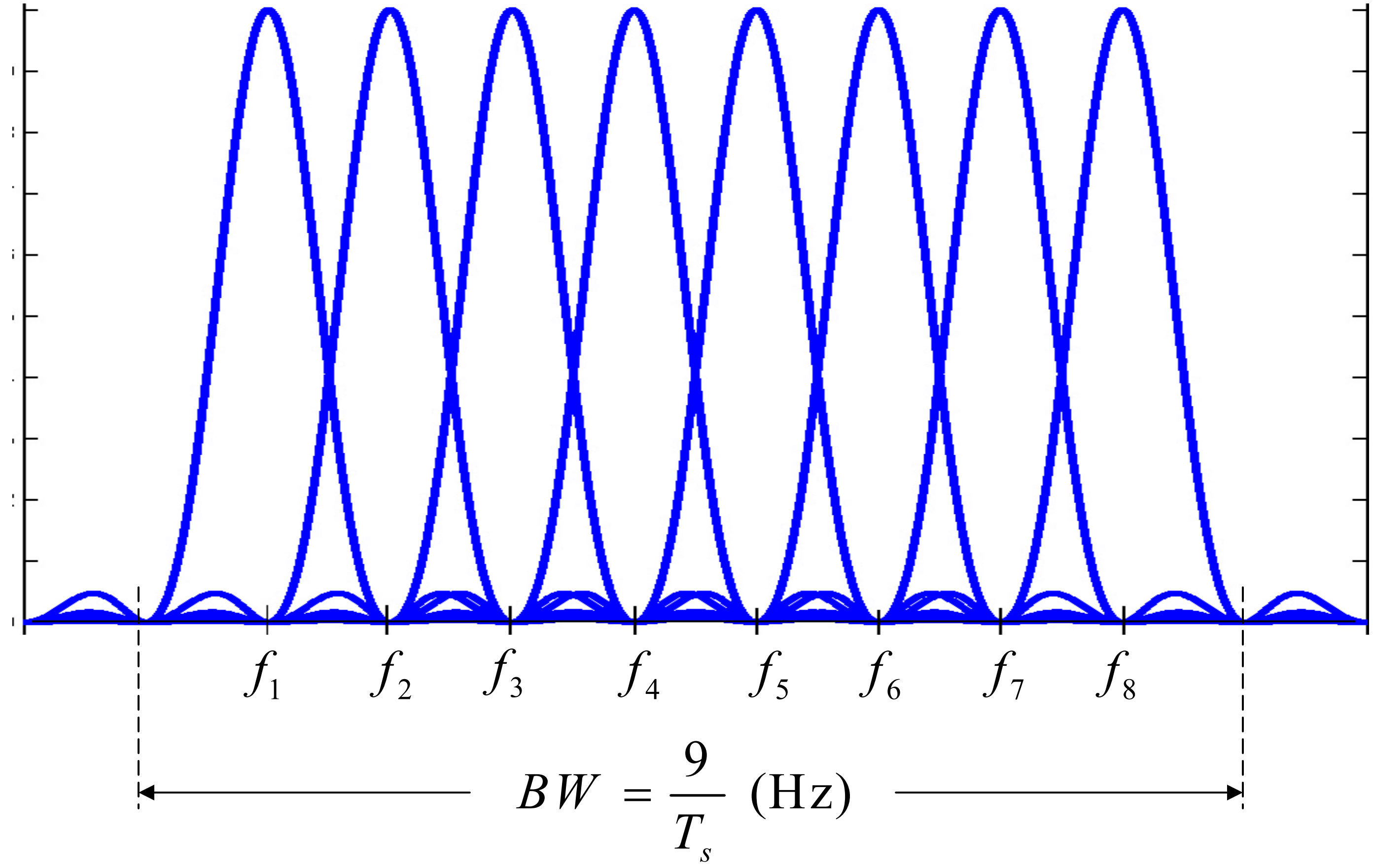
$$BW = \frac{2.5}{T_s}$$

6.1. M-Frequency Shift Keying (M-FSK) – cont.

Bandwidth Transmisi 8-FSK:

8-FSK dgn Non-Coherent Receiver

$$BW_{\text{non-coherent (M-FSK)}} = \frac{M+1}{T_s} \text{ (Hz)}$$

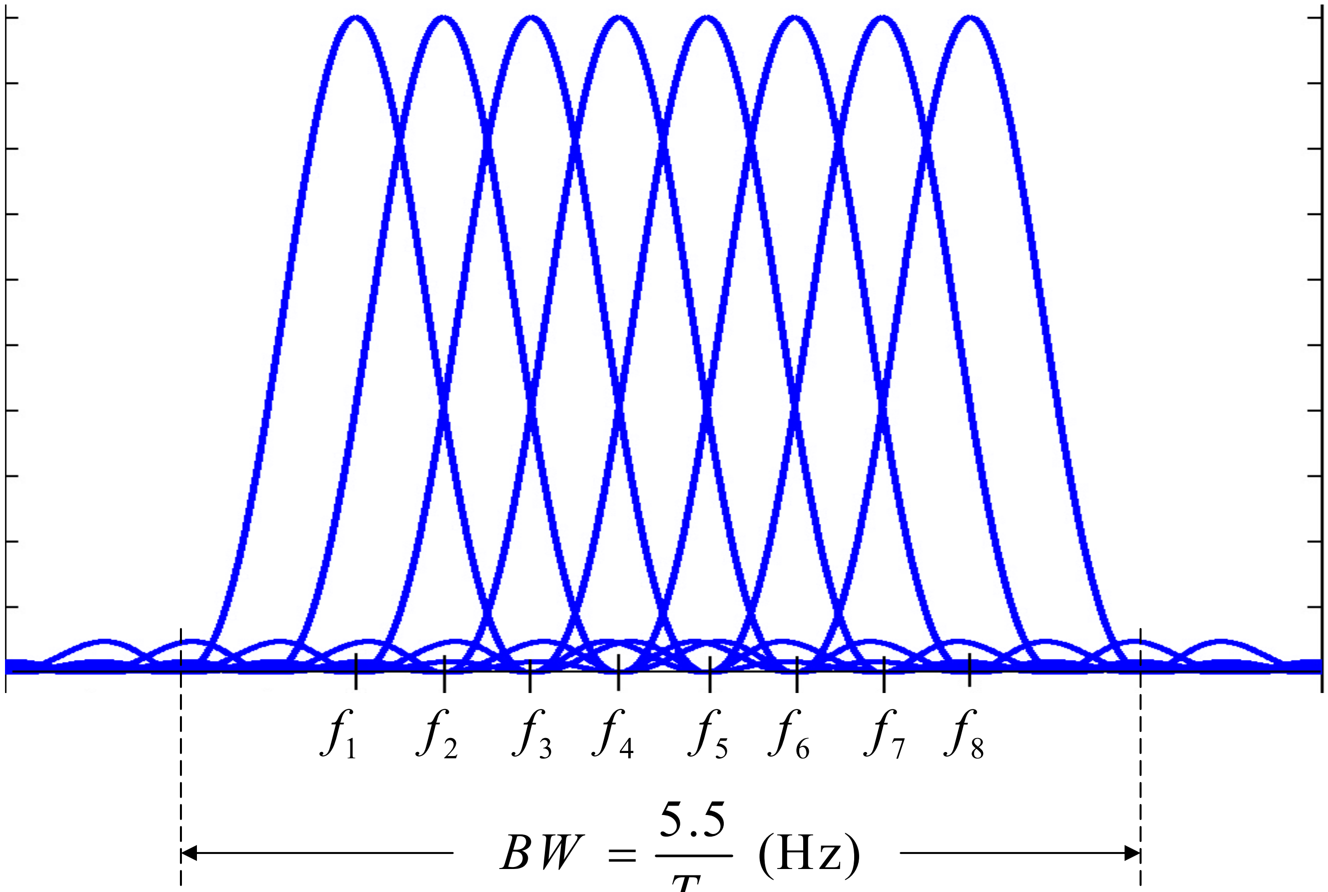


6.1. M-Frequency Shift Keying (M-FSK) – cont.

Bandwidth Transmisi 8-FSK:

$$BW_{coherent\,(M\text{-}FSK)} = \frac{M+3}{2T_s}\ \text{(Hz)}$$

8-FSK dgn Coherent Receiver

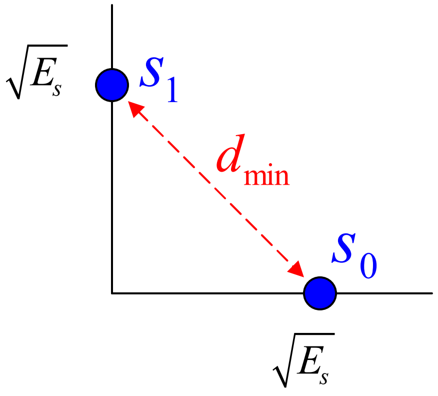


$$BW = \frac{5.5}{T_s}\ \text{(Hz)}$$

6.1. M-Frequency Shift Keying (M-FSK) – cont.

***M* = 2 (Binary FSK - BFSK)**

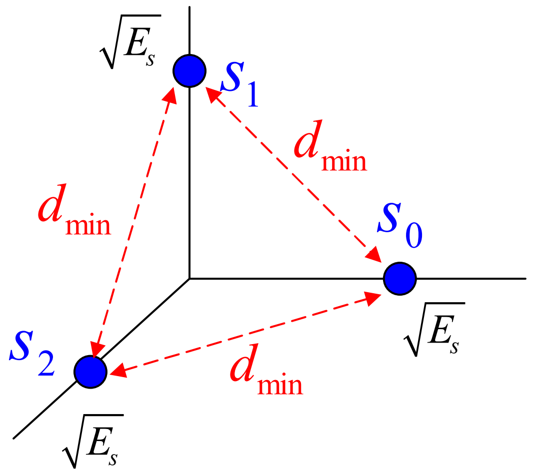
Konstelasi Sinyal:



$$\sqrt{E_s} = A\sqrt{\frac{T_s}{2}}$$

***M* = 3** *(hanya untuk ilustrasi)*

Konstelasi Sinyal:



$$\sqrt{E_s} = A\sqrt{\frac{T_s}{2}}$$

- Semua sinyal M-FSK memiliki energi (E_s) dan jarak (d_{min}) yang sama.

6.2. Probabilitas Simbol Error untuk M-FSK.

Probabilitas Simbol Error:

BFSK dengan Coherent Receiver:

$$P_{se} = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

M-FSK dengan Coherent Receiver:

$$P_{se} \leq (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

Relasi antara P_{se} (SER) dan P_{be} (BER) untuk M-FSK

$$P_{be} = \frac{M/2}{M-1} P_{se}$$

6.2. Probabilitas Simbol Error untuk MFSK – cont.

Probabilitas Simbol Error:

BFSK dengan Non-Coherent (sub-optimal) Receiver:

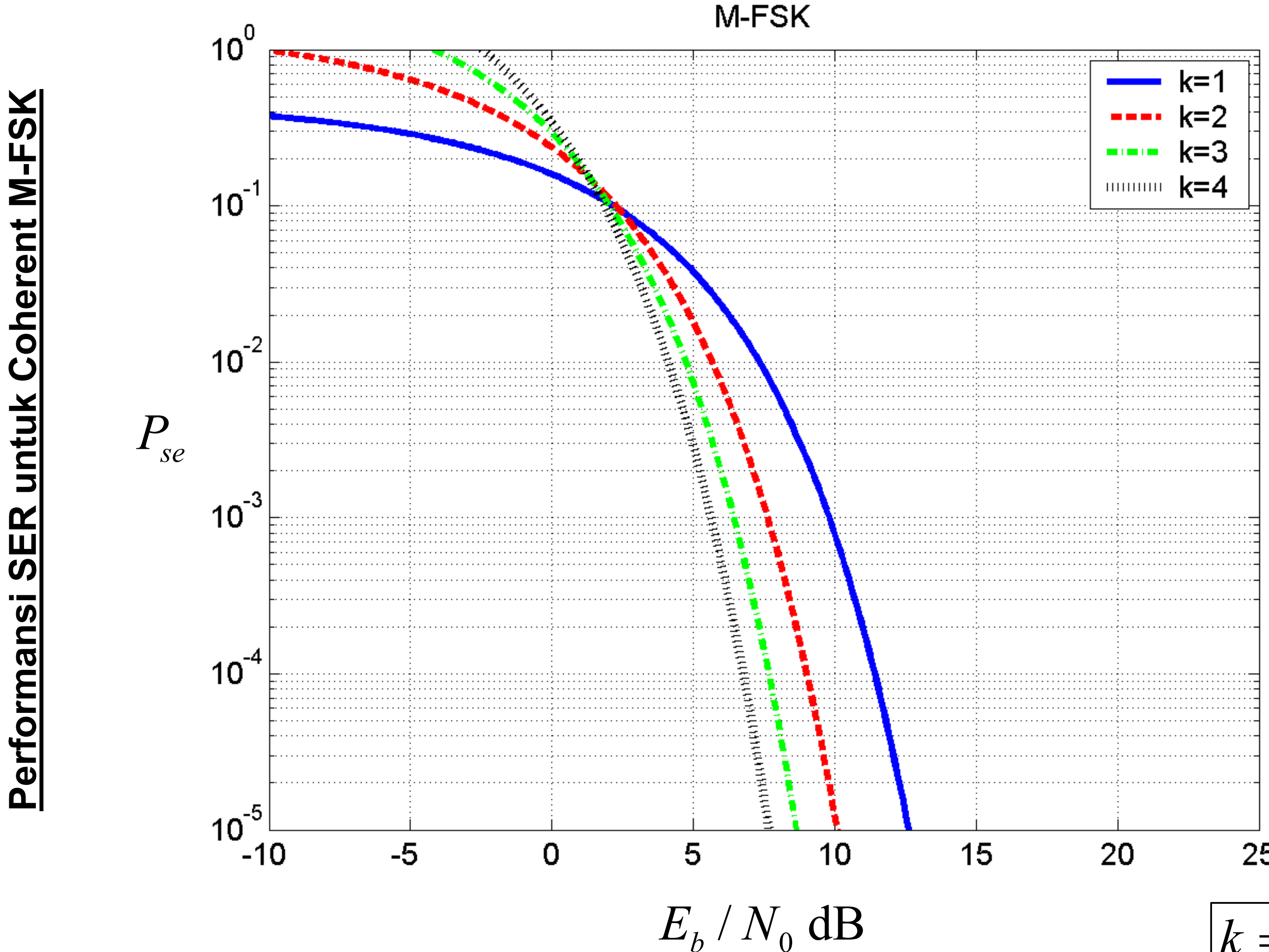
$$P_{se} = \frac{1}{2}\exp\left(-\frac{E_s}{2N_0}\right)$$

M-FSK dengan Non-Coherent (sub-optimal) Receiver:

$$P_{se} = \frac{1}{M}\exp\left(-\frac{E_s}{N_0}\right)\sum_{j=2}^{M}(-1)^j \binom{M}{j} \exp\left(\frac{E_s}{jN_0}\right)$$

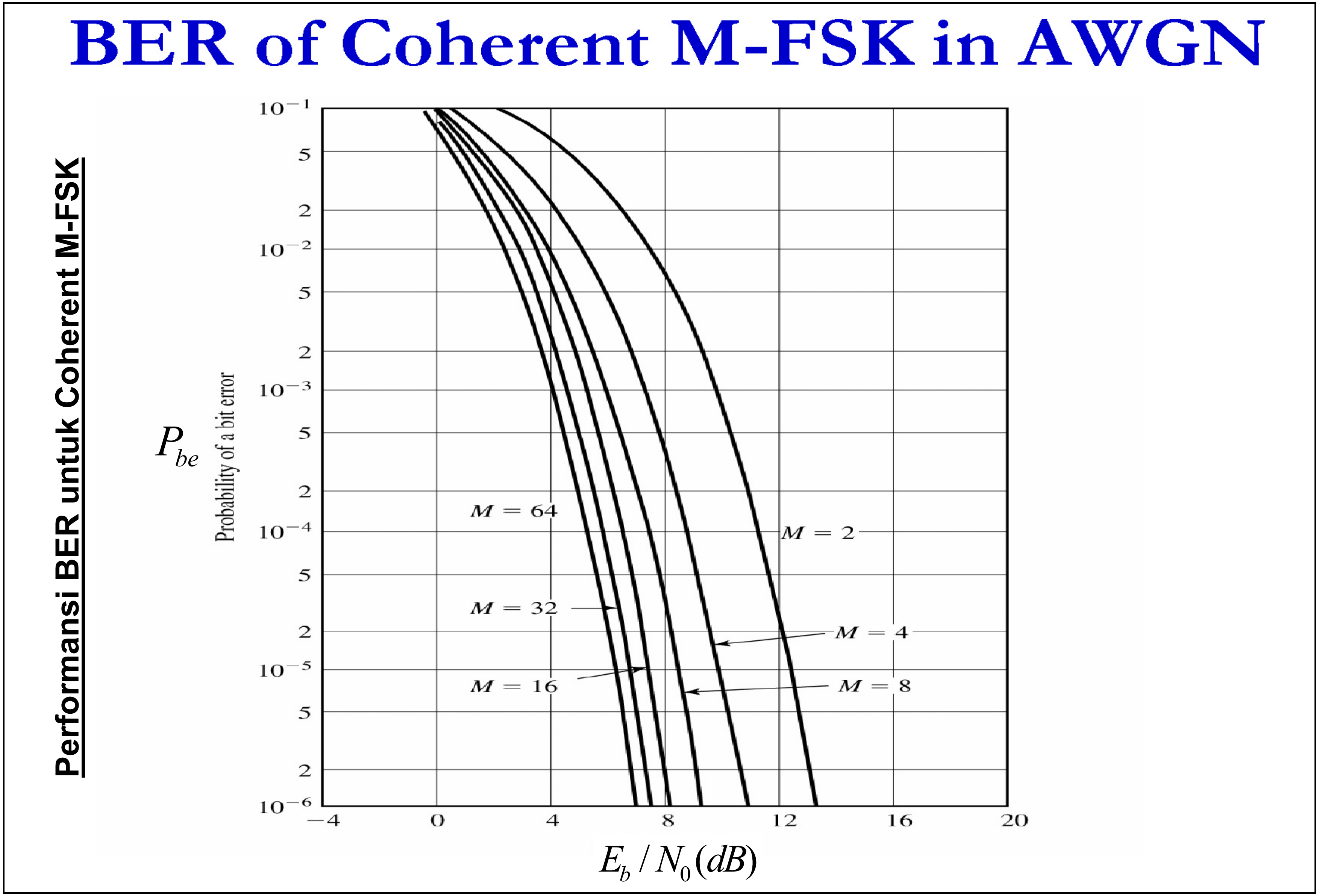
$$\text{,dimana} \quad \binom{M}{j} = \frac{M!}{j!(M-j)!}$$

6.2. Probabilitas Simbol Error untuk M-FSK – cont.



$k = \log_2 M$

6.2. Probabilitas Simbol Error untuk MFSK – cont.



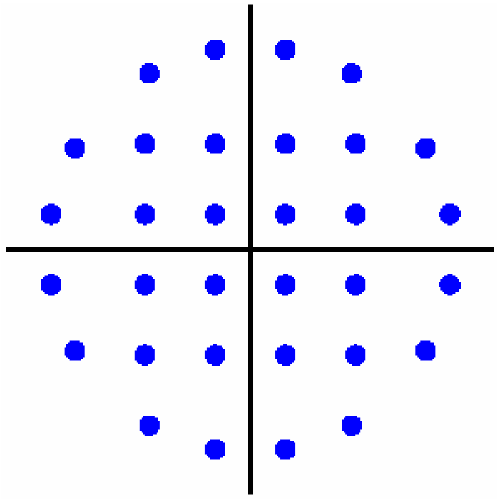
6.3. M-Qudrature Amplitude Modulation (M-QAM)

Modulasi QAM:

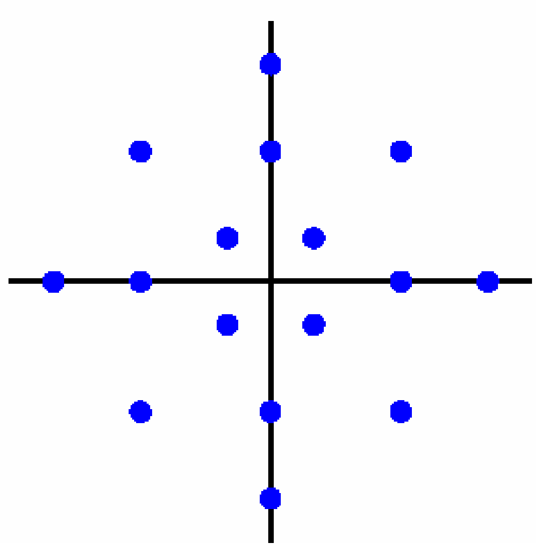
$$m_i \longrightarrow s_i(t) = \sqrt{\frac{2E_i}{T_s}}\cos\left(\omega_c t + \theta_i\right) \quad ; 0 \leq t \leq T_s$$

$$; i = 0, 2, \ldots, M\text{-}1$$

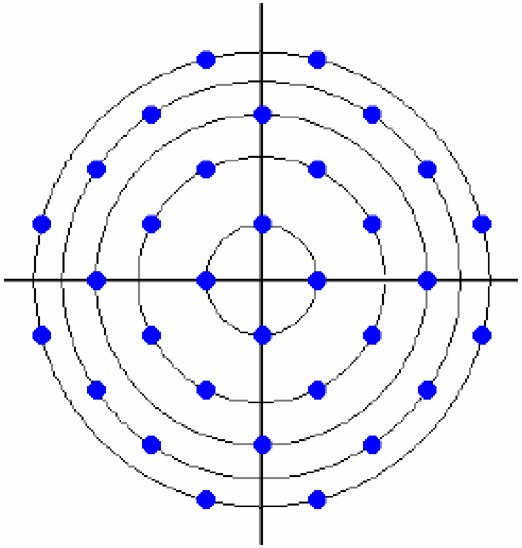
Beberapa contoh konstelasi sinyal QAM:



$M = 32$



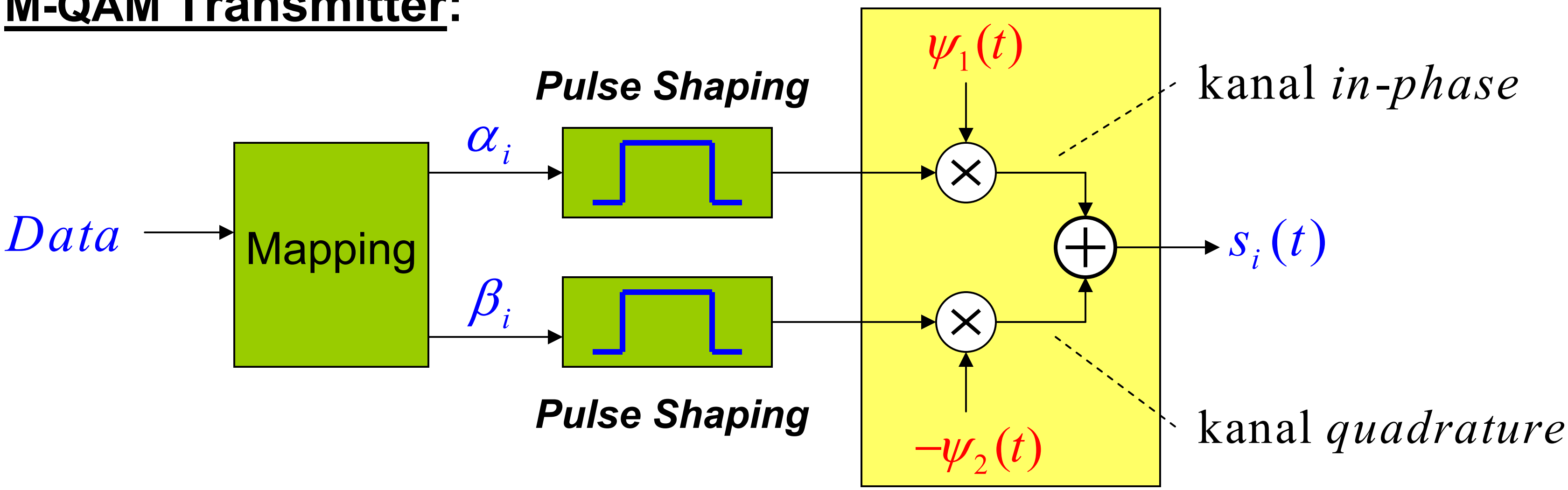
$M = 16$



$M = 32$

6.3. M-Quadrature Amplitude Modulation (M-QAM) – cont.

M-QAM Transmitter:



$$s_i(t) = \sqrt{\frac{2E_i}{T_s}} \cos\left(\omega_c t + \theta_i\right) \qquad ;\ 0 \le t \le T_s \quad ;\ i = 0, 2, \ldots, M\text{-}1$$

$$= \underbrace{\sqrt{E_i}\cos(\theta_i)}_{\alpha_i}\underbrace{\sqrt{\frac{2}{T_s}}\cos(\omega_c t)}_{\psi_1(t)} - \underbrace{\sqrt{E_i}\sin(\theta_i)}_{\beta_i}\underbrace{\sqrt{\frac{2}{T_s}}\sin(\omega_c t)}_{\psi_2(t)}$$

Komponen *in-phase* Komponen *quadrature*

6.3. M-Quadrature Amplitude Modulation (M-QAM) – cont.

Representasi Quadrature:

$$s_i(t) = A_i \cos\left(\omega_c t + \theta_i\right) \qquad ; 0 \le \mathrm{t} \le \mathrm{T_s} \quad ; \mathrm{i} = 0, 2, \ldots, \mathrm{M\text{-}1}$$

$$= \frac{A_i}{2}\left\{e^{j(\omega_c t+\theta_i)} + e^{-j(\omega_c t+\theta_i)}\right\}$$

$$= \frac{A_i}{2}\left\{e^{j(\omega_c t)}e^{j(\theta_i)} + e^{-j(\omega_c t)}e^{-j(\theta_i)}\right\}$$

$$A_i = \sqrt{\frac{2E_i}{T_s}}$$

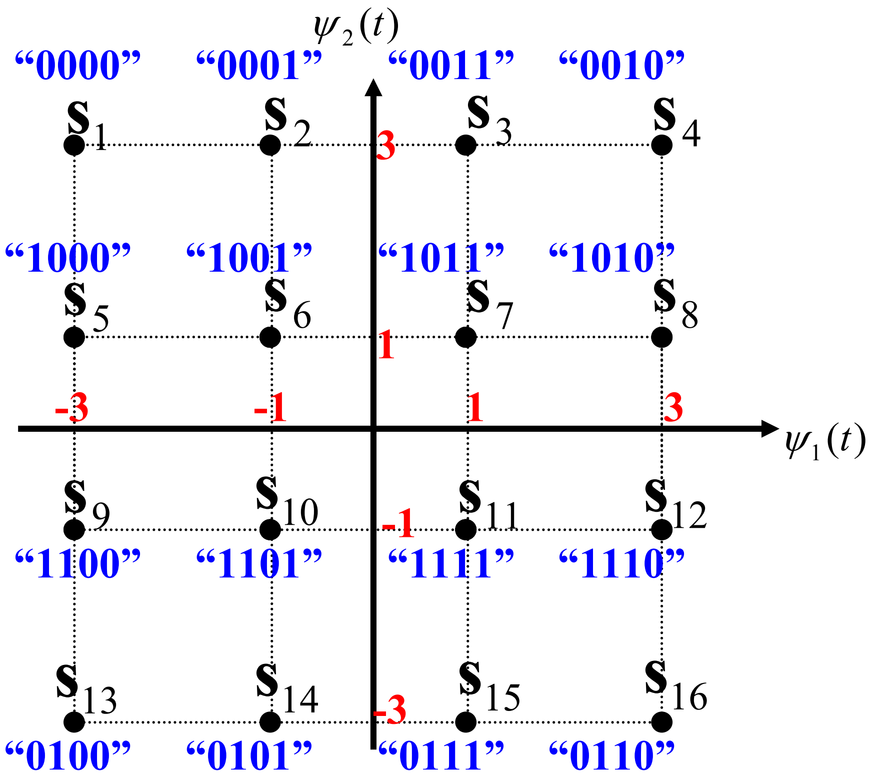
$$= \frac{A_i}{2}\left\{\left[\cos(\omega_c t) + j\sin(\omega_c t)\right]\left[\cos(\theta_i) + j\sin(\theta)\right] + \left[\cos(\omega_c t) - j\sin(\omega_c t)\right]\left[\cos(\theta_i) - j\sin(\theta)\right]\right\}$$

$$= A_i\left\{\cos(\theta_i)\cos(\omega_c t) - \sin(\theta_i)\sin(\omega_c t)\right\}$$

$$= \sqrt{E_i}\cos(\theta_i)\sqrt{\frac{2}{T_s}}\cos(\omega_c t) - \sqrt{E_i}\sin(\theta_i)\sqrt{\frac{2}{T_s}}\sin(\omega_c t)$$

6.3. M-Quadrature Amplitude Modulation (M-QAM) – cont.

Contoh: 16-QAM (Konstelasi Kubus)

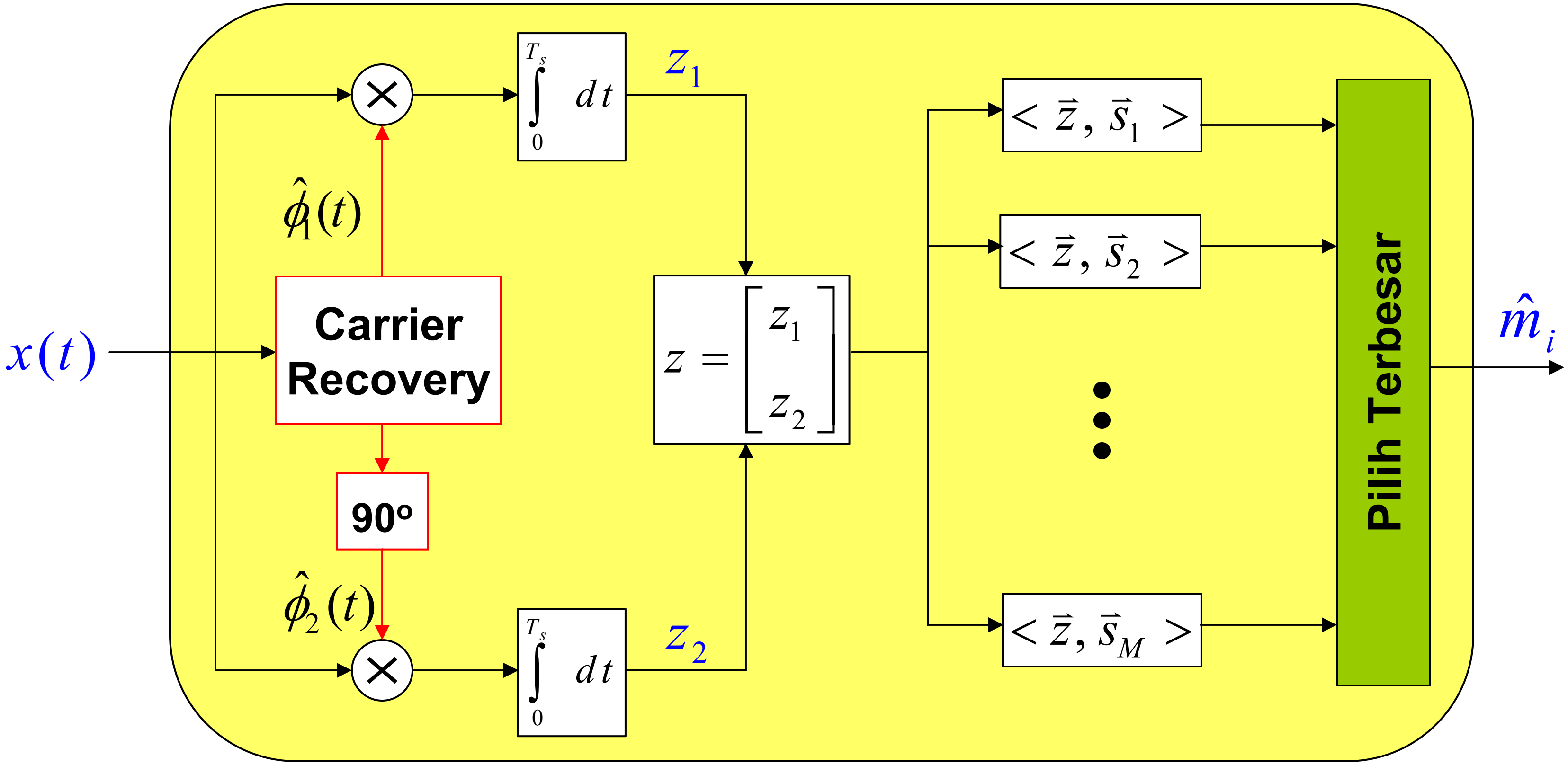


$$\psi_1(t) = \sqrt{2/T_s}\cos(\omega_c t)$$

$$\psi_2(t) = -\sqrt{2/T_s}\sin(\omega_c t)$$

5.2. M-Phase Shift Keying (M-PSK) – cont.

M-QAM Coherent Receiver dgn MED Detection:

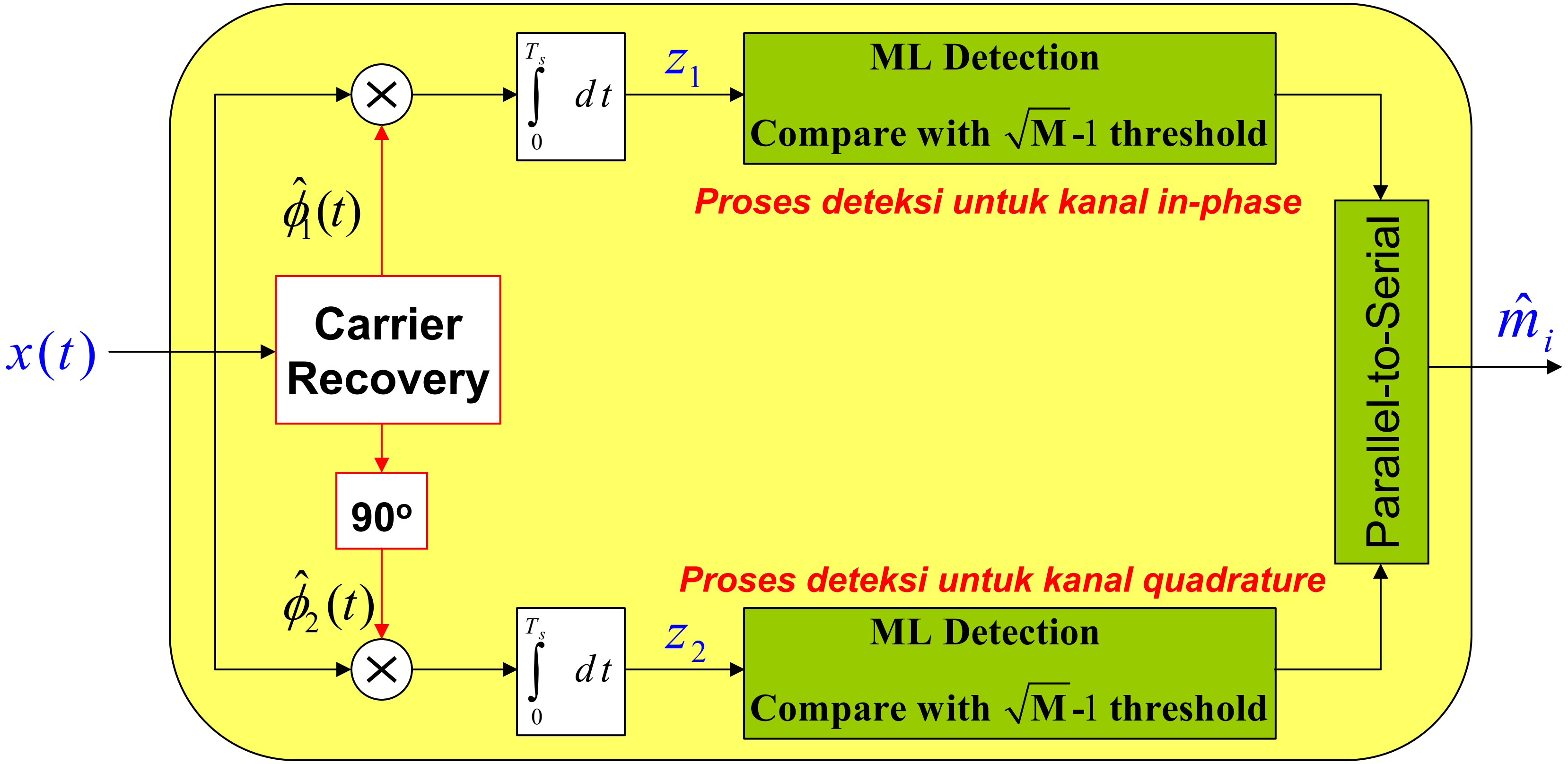


$$\hat{\phi}_1(t) = \sqrt{2/T_s}\,\cos(\omega_c t + \hat{\alpha})$$

$$\hat{\phi}_2(t) = -\sqrt{2/T_s}\,\sin(\omega_c t + \hat{\alpha})$$

5.2. M-Phase Shift Keying (M-PSK) – cont.

M-QAM Coherent Receiver dgn ML Detection:



$$\hat{\phi}_1(t) = \sqrt{2/T_s}\cos(\omega_c t + \hat{\alpha})$$

$$\hat{\phi}_2(t) = -\sqrt{2/T_s}\sin(\omega_c t + \hat{\alpha})$$

6.4. Probabilitas Simbol Error untuk M-QAM.

Probabilitas Simbol Error:

M-QAM (konstelasi kubus) dengan Coherent Receiver:

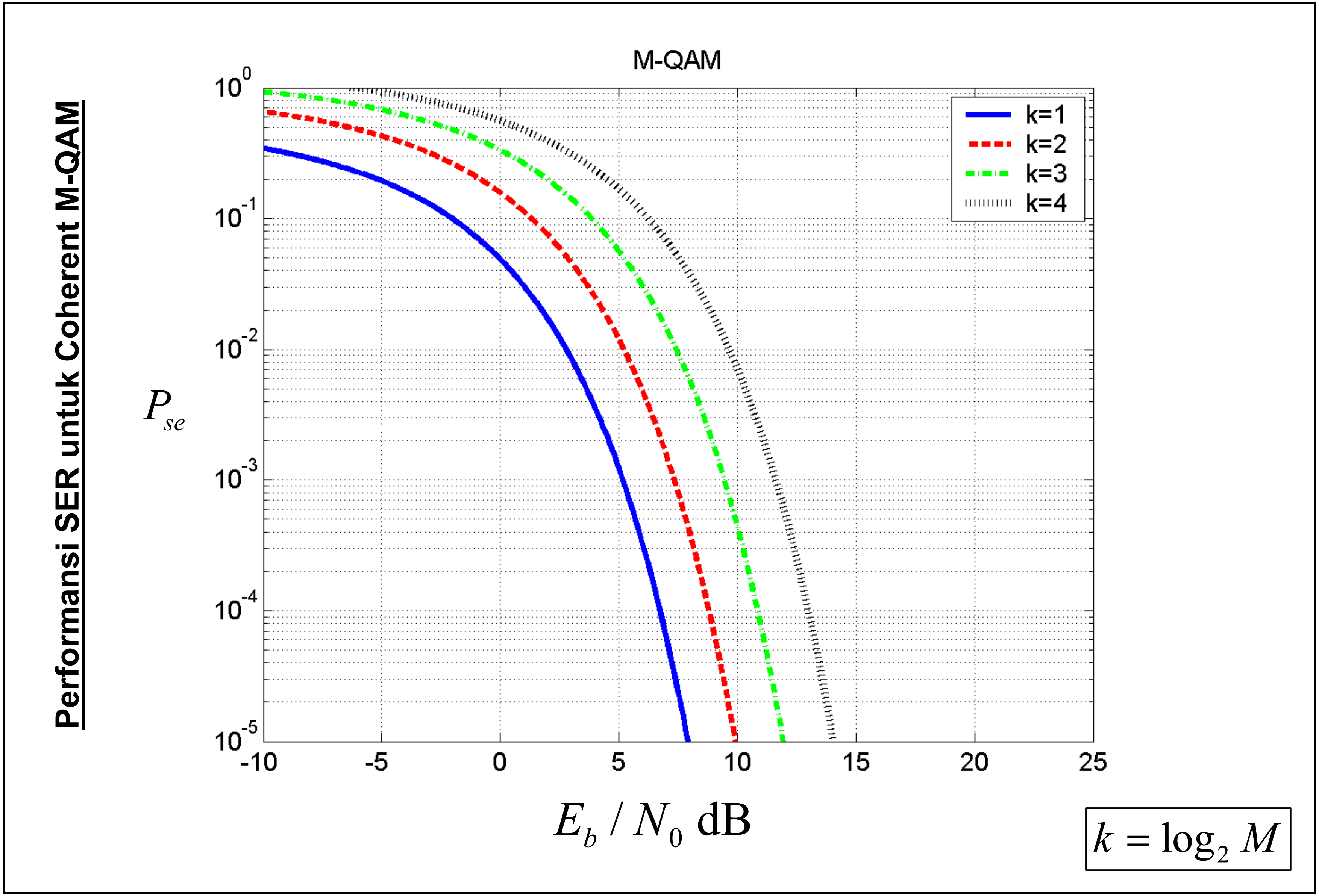
$$P_{se} = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_0}}\right)$$

$$E_s = \log_2 M \cdot E_b$$

Relasi antara P_{se} (SER) dan P_{be} (BER) untuk M-QAM

$$P_{be} \approx \frac{P_{se}}{\log_2 M}$$

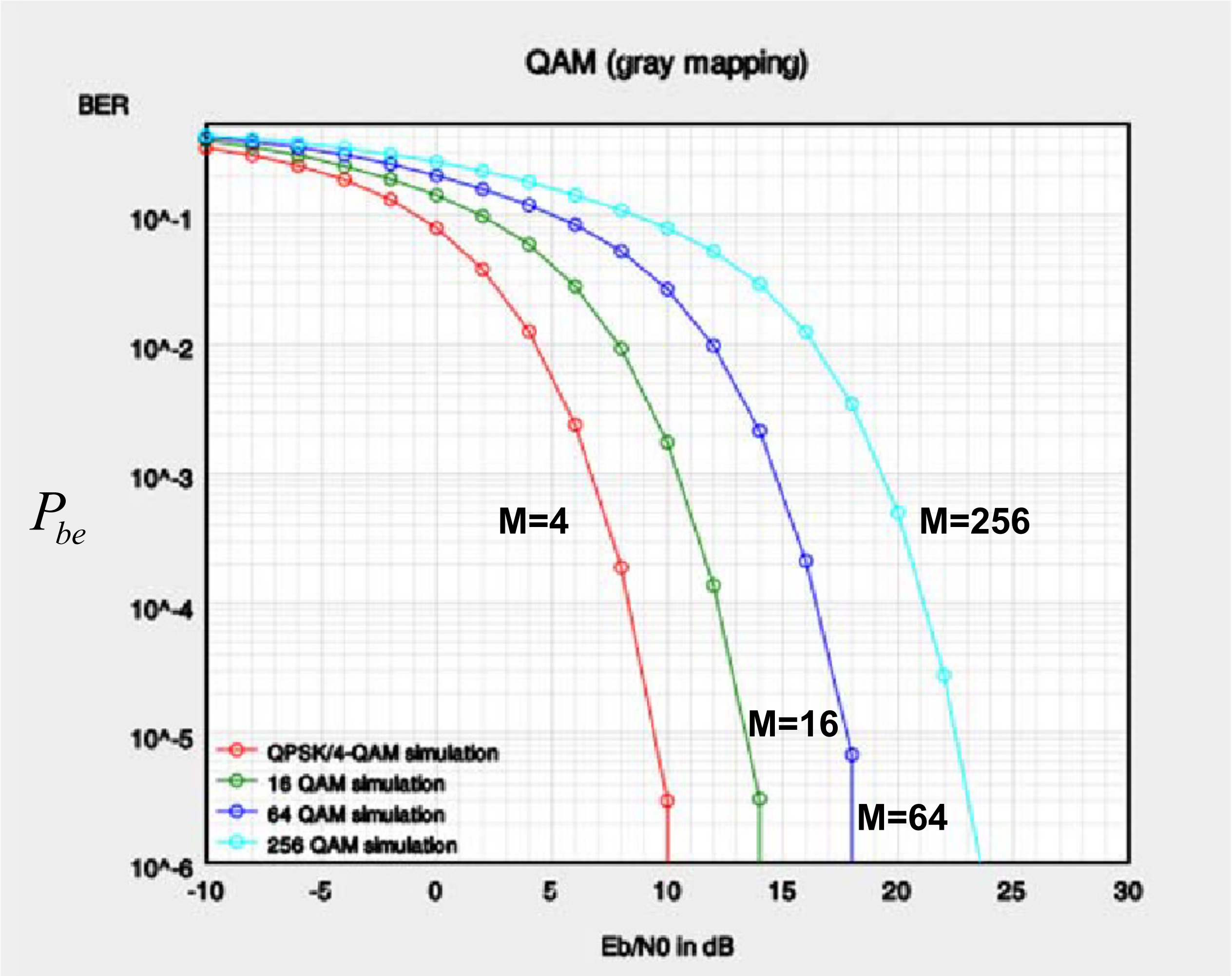
6.4. Probabilitas Simbol Error untuk M-QAM – cont.



6.4. Probabilitas Simbol Error untuk M-QAM – cont.

BER of Coherent M-QAM in AWGN

Performansi BER untuk Coherent M-QAM

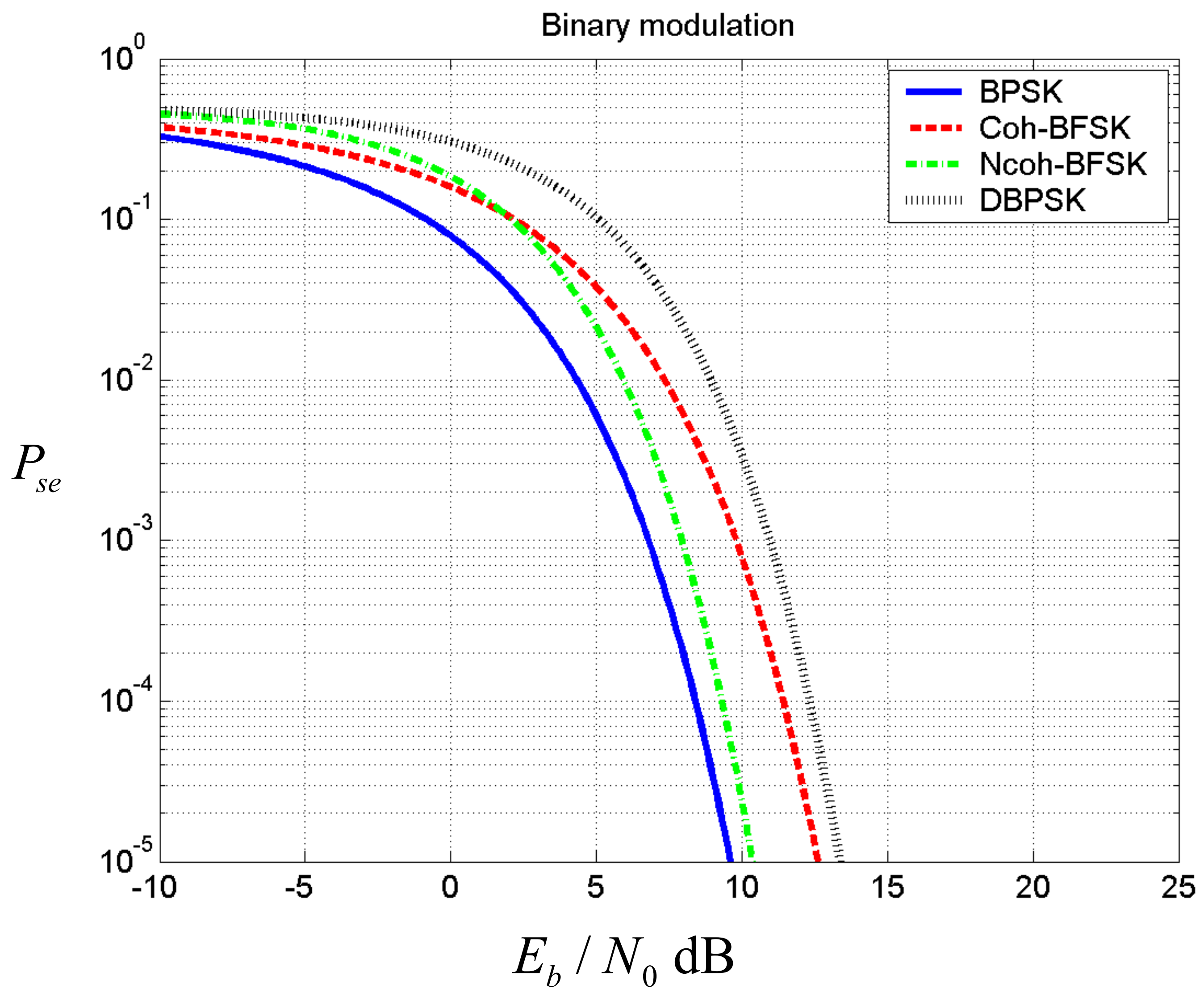


6.5. Perbandingan antara M-PSK, M-FSK, & M-QAM.

Binary PSK (BPSK) vs. Binary DPSK vs. Binary FSK (BFSK)			
Pulse Shape: Raised Cosine (β=0)	BER	Bandwidth	Bandwidth Efficiency
	P_{be}	BW	$\eta = R_b / BW$
BPSK$_{coherent}$	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$\frac{1}{T_b}$ (Hz)	1 (bit/s/Hz)
Binary DPSK	$\frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$	$\frac{1}{T_b}$ (Hz)	1 (bit/s/Hz)
BFSK$_{coherent}$	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$	$\frac{3}{2T_b}$ (Hz)	0.67 (bit/s/Hz)
BFSK$_{non-coherent}$	$\frac{1}{2}\exp\left(-\frac{1}{2}\frac{E_b}{N_0}\right)$	$\frac{2}{T_b}$ (Hz)	0.5 (bit/s/Hz)

6.5. Perbandingan antara M-PSK, M-FSK, & M-QAM - cont.

Performansi SER untuk BPSK, BFSK, dan DPSK



6.5. Perbandingan antara M-PSK, M-FSK, & M-QAM - cont.

M-PSK vs. M-FSK vs. M-QAM			
Pulse Shape: Raised Cosine (β=0)	BER	Bandwidth	Bandwidth Efficiency
	P_{be}	BW	$\eta = R_b / BW$
M-PSK$_{\text{coherent}}$	$\frac{2}{\log_2 M} Q\left(\sqrt{2\log_2 M \cdot \frac{E_b}{N_0}} \sin\frac{\pi}{M}\right)$	$\frac{1}{\log_2 M \cdot T_b}$	$\log_2 M$
M-DPSK	$\approx 2 \cdot P_{be(M-PSK)}$	$\frac{1}{\log_2 M \cdot T_b}$	$\log_2 M$
M-FSK$_{\text{coherent}}$	$\leq \frac{M}{2} Q\left(\sqrt{\log_2 M \cdot \frac{E_b}{N_0}}\right)$	$\frac{M+1}{2\log_2 M \cdot T_b}$	$\frac{2\log_2 M}{M+1}$
M-QAM$_{\text{coherent}}$	$\frac{4}{\log_2 M}\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\log_2 M}{M-1} \frac{E_b}{N_0}}\right)$	$\frac{1}{\log_2 M \cdot T_b}$	$\log_2 M$

6.5. Perbandingan antara M-PSK, M-FSK, & M-QAM - cont.

Ringkasan:

> *Bertambah besar M:*

► *M-FSK:* - *bertambah tinggi bit-ratenya.*
- *bertambah besar bandwidth transmisinya.*
- *bertambah baik performansi BER-nya.*

► *M-PSK:* - *bertambah besar bit-ratenya (bandwidth tetap).*
- *bertambah buruk performansi BER-nya.*

► *M-QAM:* - *bertambah besar bit-ratenya (bandwidth tetap).*
- *bertambah buruk performansi BER-nya.*

> *Bandwidth Effciency*

► *M-QAM dan M-PSK memiliki bandwidth efficiency yang setara, tetapi untuk nilai Eb/No yang sama performansi BER M-QAM lebih baik dari M-PSK (dan bertambah lebih baik dengan mem-besarnya M).*

► *M-FSK memiliki bandwidth efficiency yang lebih buruk dibanding M-PSK dan M-QAM.*